Dividing Polynomials

Recall that when a numerical division has a remainder of zero, as shown below, the divisor and quotient are both factors of the dividend.

$$\begin{array}{r} 7 \\ 8\overline{)56} \\ \underline{56} \\ 0 \end{array}$$

7 and 8 are factors of 56.

If numerical division leaves a nonzero remainder, as shown below, then neither the divisor nor the quotient is a factor of the dividend.

$$\begin{array}{r} 8 \\ 5\overline{)42} \\ \underline{40} \\ 2 \end{array}$$

Neither 5 nor 8 is a factor of 42.

The same is true for polynomial division. If you divide a polynomial by one of its factors, then you get another factor. When a polynomial division leaves a zero remainder, as shown below with monomials, you have factored the polynomial.

$$\begin{array}{r} 2x \\ x\overline{)2x^2} \\ \underline{2x^2} \\ 0 \end{array}$$

x and $2x$ are factors of $2x^2$.

Example 1: Long Division

Divide $x^2 + 3x - 12$ by $x - 3$.

$$x + 6$$

$$x - 3 \,\big)\, x^2 + 3x - 12$$

$$- \quad x^2 - 3x$$

$$0 + 6x - 12$$

$$- \quad 6x - 18$$

$$0 + 6$$

$$\boxed{x + 6 \text{ R } 6}$$

Neither $x-3$ or $x+6$ are factors of $x^2+3x-12$.

Example 2: Long Division

Divide $x^2 - 3x + 1$ by $x - 4$.

$$
\begin{array}{r|l}
 & \;\; x + 1 \\
\hline
x - 4 & x^2 - 3x + 1 \\
+ & -x^2 + 4x \downarrow \\
\hline
 & 0 + x + 1 \\
- & \qquad x - 4 \\
\hline
 & 0 + 0 + 5
\end{array}
$$

$x + 1$ R 5

The remandar 5.

Example 3: Synthetic Division

Use synthetic division to divide $3x^3 - 4x^2 + 2x - 1$ by $x + 1$.

$$x+1 \overline{\big)\, 3x^3 - 4x^2 + 2x - 1}$$

$$\begin{array}{r|rrrr} -1 & 3 & -4 & 2 & -1 \\ \times & \downarrow & +\,-3 & +\,7 & +\,-9 \\ \hline & 3 & -7 & 9 & -10 \end{array}$$

Quad Linear C R

$3x^2 - 7x + 9 \; R -10$

Example 4: Synthetic Division

Divide using synthetic division.

$$(3x^3 + 17x^2 + 21x - 9) \div (x + 3)$$

$$\begin{array}{r|rrrr}
-3 & 3 & 17 & 21 & -9 \\
 & \downarrow & + & + & + \\
 & & -9 & -24 & 9 \\
\hline
 & 3 & 8 & -3 & 0 \\
 & Q & L & C & R
\end{array}$$

$3x^2+8x-3$ and $x+3$

are factors of $3x^3+17x^2+21x+9$.

$(x^3+27) \div (x+3)$

$(x^3+0x^2+0x+27) \div (x+3)$

$$
\begin{array}{r|rrrr}
-3 & 1 & 0 & 0 & 27 \\
 & \downarrow & + & + & + \\
 & & -3 & 9 & -27 \\
\hline
 & 1 & -3 & 9 & 0 \\
 & Q & L & C & R
\end{array}
$$

x^2-3x+9 and $x+3$ are factors of x^3+27.