Test Practice 1: Rates, Averages, and Proportions

Estimated time: 45 minutes

Directions: Read each question. Choose the best answer or write the answer to the question in the space you are given.

1 The coach wants to find the mean number of yards run. He has found the sum of the yards. What does he have to do next to find the mean?

Yards Run					
9	12	5	7	6	4
8	21	15	11	12	8

Ⓐ Find the middle number.
Ⓑ Divide by 2.
Ⓒ Subtract 21 from 12.
Ⓓ Divide by 12.

2 Solve for n.

$$\frac{9}{24} = \frac{n}{16}$$

Ⓐ 6 Ⓒ 19
Ⓑ 13.5 Ⓓ 42.7

3 The density of a diamond is 3.52 grams per cubic centimeter. Sally has a diamond that has a volume of 20 mm^3. Which shows the weight of the diamond?

Ⓐ 70.4 g Ⓒ 0.704 g
Ⓑ 7.04 g Ⓓ 0.0704 g

4 The ratio of fifth graders to sixth graders in a science club is 4 to 7. There are 20 fifth graders. How many sixth graders are there?

Ⓐ 11 Ⓒ 35
Ⓑ 28 Ⓓ 70

5 How far can you bicycle in 3 hours if you achieve an average speed of 15 mi/hr?

Ⓐ 5 mi Ⓒ 30 mi
Ⓑ 18 mi Ⓓ 45 mi

6 Each science project is given a score from 0 to 100 by each judge. The teacher's score counts twice as much as each of the three student judge's scores. Here are Marge's scores on her science project.

Student judges: 80, 75, 78
Teacher judge: 82

What is Marge's final score?

Ⓐ 63 Ⓒ 79.4
Ⓑ 78.75 Ⓓ 99.25

7 Suppose it takes 480 person-hours to complete a job. How many hours will 6 workers need to work to do the job?

Ⓐ 8 hours Ⓒ 80 hours
Ⓑ 40 hours Ⓓ 48 hours

8 Philip can run a mile in 6 minutes. What is his speed in miles per hour?

Ⓐ 6 mi/hr Ⓒ 12 mi/hr
Ⓑ 10 mi/hr Ⓓ 60 mi/hr

GO ON

NO WORK, NO CREDIT

Computing with Rational Numbers

9 Find the quotient.

$$\frac{8}{15} \div \left(-\frac{2}{5}\right) =$$

Ⓐ $\frac{16}{75}$ Ⓒ $1\frac{1}{3}$

Ⓑ $-\frac{16}{75}$ Ⓓ $-1\frac{1}{3}$

Dividing by Fractions

To divide by a fraction, multiply by the reciprocal of the divisor.

- The reciprocal of $\frac{3}{16}$ is $\frac{16}{3}$.
- The reciprocal of 5 is $\frac{1}{5}$.
- The reciprocal of $1\frac{1}{4}$ is $\frac{4}{5}$.

10 Find the sum.

$$-\frac{1}{10} + \frac{5}{6} =$$

Ⓐ $\frac{11}{15}$ Ⓒ $\frac{14}{15}$

Ⓑ $-\frac{11}{15}$ Ⓓ $-\frac{14}{15}$

Remember

When adding or subtracting fractions, you must first find a common denominator. Then add or subtract the numerators.

Step-By-Step

In **example 9** you are asked to divide rational numbers. **Rational numbers** are numbers that can be written as the ratio of two integers.

1 Divide as if the numbers were positive fractions.

$$\frac{8}{15} \div \frac{2}{5} = \frac{8}{15} \times \frac{5}{2} =$$

2 Use the rules for dividing integers. The signs are different, so the quotient is negative.

$$\frac{8}{15} \div \left(-\frac{2}{5}\right) =$$

3 Simplify by changing the fraction to a mixed number.

$$-\frac{4}{3} =$$

Step-By-Step

For **example 10**, begin by finding a common denominator.

1 What is the least common multiple of 10 and 6?

$$\text{LCM}(10, 6) =$$

2 Use the LCM, 30, to write equivalent fractions.

$$\frac{1}{10} \times \frac{3}{3} = \frac{3}{30} \qquad \frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$$

3 Add the numerators. Simplify.

$$-\frac{3}{30} + \frac{25}{30} = \qquad =$$

NO WORK, NO CREDIT

Estimating with Rational Numbers

11 Choose the best estimate for this quotient.

$$3.507 \div 0.028 \approx$$

Ⓐ 1.2 Ⓒ 120

Ⓑ 12 Ⓓ 1,200

Remember

The symbol $\approx$ means
"is approximately equal to."

Step-By-Step

For **example 11**, use compatible numbers. Round the two numbers in the problem so that you can do the computation more easily.

1 Round the divisor first. Round to the nearest hundredth.

0.028 rounds to

2 Round the dividend to 3.6 so the computation will be easy.

$3.6 \div 0.03 =$

$3.507 \div 0.028 \approx$

12 Choose the best estimate for this sum.

$$4\tfrac{9}{16} + 2\tfrac{5}{8} \approx$$

Ⓐ less than 6

Ⓑ between 6 and 7

Ⓒ between 7 and 8

Ⓓ greater than 8

Remember

A fraction is greater than $\frac{1}{2}$ if the numerator is greater than one half the denominator. These fractions are greater than $\frac{1}{2}$.

$\frac{3}{4}$ $\frac{5}{8}$ $\frac{7}{12}$ $\frac{8}{15}$ $\frac{13}{25}$

Step-By-Step

For **example 12**, compare the fractional parts of the mixed numbers with $\frac{1}{2}$.

1 Both fractional parts are greater than $\frac{1}{2}$, so their sum is greater than 1.

2 Add the whole number parts. Then add 1 to that sum to estimate the sum of the fractional parts.

$4\frac{9}{16} + 2\frac{5}{8} \approx$

3 To figure out if the estimate is less than or greater than 7, think:

$4\frac{1}{2} + 2\frac{1}{2} = 7$

so—

$4\frac{9}{16} + 2\frac{5}{8}$ must be greater than 7.

GO ON

NO WORK, NO CREDIT

MATH Lesson 3

Algebra: Expressions and Equations

Properties of Real Numbers
Computing with Expressions
Solving Equations
Applying Equations

Examples 1–13: Read each question. Choose the best answer or write the answer to the question in the space you are given.

Properties of Real Numbers

1 Which number sentence illustrates the Commutative Property?

Ⓐ $7 + 6 = 8 + 5$
Ⓑ $(4 + 8) + 10 = (5 + 7) + 10$
Ⓒ $15 + (10 + 25) = 15 + (25 + 10)$
Ⓓ $15 + (10 + 25) = (15 + 10) + 25$

Step-By-Step

For **example 1**, review the basic real number properties shown at the bottom of this page.

1 Read the definition for the Commutative Property of Addition. Which answer choices have the same numbers on each side of the equation?

2 Which answer changes the order of the numbers without changing the way they are grouped?

2 Fill the blank with a number that will show the Associative Property of Multiplication.

$18 \bullet (42 \bullet 36) = (18 \bullet 42) \bullet$ ______

Step-By-Step

Review the *Associative Property* in the chart below. It states that you can group factors in different ways without changing the product.

Commutative Property	Associative Property	Distributive Property
The order in which you **add** or **multiply** does not change the answer	You can group **addends** or **factors** in different ways	The product of a sum equals the sum of two products
$12 + 37 = 37 + 12$	$(14 + 7) + 6 = 14 + (7 + 6)$	$3(12 + 8) = (3 \times 12) + (3 \times 8)$
$2.1 \bullet 6 = 6 \bullet 2.1$	$(5 \times 2) \times 8 = 5 \times (2 \times 8)$	$(\frac{1}{2} \bullet 14) + (\frac{1}{2} \bullet 10) = \frac{1}{2}(14 + 10)$

NO WORK, NO CREDIT

Properties of Real Numbers

3 Use properties of operations and mental math to solve this problem.

$$25 + (36 + 75) \times 1 =$$

Answer:________________

4 What is the reciprocal of 0.75?

Ⓐ $1\frac{5}{7}$

Ⓑ $1\frac{1}{3}$

Ⓒ $1\frac{1}{4}$

Ⓓ $1\frac{3}{4}$

Opposites and Reciprocals

When you add $+5$ to -5, you get 0. The numbers $+5$ and -5 are opposites. **Opposites** are the same distance from 0 on a number line.

When you multiply $\frac{2}{3}$ times $\frac{3}{2}$, you get $\frac{6}{6}$, which equals 1. The numbers $\frac{2}{3}$ and $\frac{3}{2}$ are **reciprocals**.

Step-By-Step

Before solving **example 3**, review the four properties in the chart at the bottom of this page.

1 First use the Identity Property of Multiplication. Multiplying by 1 will not change the sum of $(36 + 75)$.

2 Use the Commutative and Associative Properties to simplify the addition.

$$25 + (36 + 75) = (25 + 75) + 36$$

3 Use mental math to add.

$$(25 + 75) + 36 = \quad$$

Step-By-Step

For **example 4**, apply the Inverse Property of Multiplication.

1 Write 0.75 as a fraction in lowest terms.

$$0.75 = \frac{75}{100} = \quad$$

2 Interchange the numerator and denominator to find the reciprocal.

The reciprocal of $\frac{3}{4}$ is ____ .

3 Change the fraction to a mixed number.

$$\frac{4}{3} = \quad$$

Property	Definition	Example
Identity Property of Addition	The sum of any number and 0 is the original number.	$x + 0 = x$
Identity Property of Multiplication	The product of any number and 1 is the original number.	$y \cdot 1 = y$
Inverse Property of Addition	The sum of any number and its opposite is 0.	$x + (-x) = 0$
Inverse Property of Multiplication	The product of any number and its reciprocal is 1.	$y \cdot \frac{1}{y} = 1$

GO ON

NO WORK, NO CREDIT

Computing with Expressions

5 Find the sum.

$(2x + 8) + (-5x + 3) =$ ________

Ⓐ $-7x + 11$
Ⓑ $3x + 5$
Ⓒ $-3x + 11$
Ⓓ $10x - 2$

Step-By-Step

For **example 5**, add expressions by adding like terms. Like terms have the same variables of the same degree. You can add horizontally or vertically.

1 To add horizontally, first add the x-terms.

$$(2x + 8) + (-5x + 3) =$$

$$2x \quad + (-5x) \quad =$$

2 Add the constant terms.

$$(2x + 8) + (-5x + 3) = -3x +$$

6 Subtract.

$(5a - 11) - (12a + b - 6) =$ ______

Ⓐ $7a - b - 5$
Ⓑ $-7a - b - 5$
Ⓒ $7a - b + 5$
Ⓓ $-7a + b - 17$

Algebraic Terms

expression: variables, numbers, and symbols that express a numerical relationship

variable: a letter that represents a number or that can represent various numbers

constant: a letter or number that stands for a fixed number

term: in the expression $2x + y$, $2x$ and y are the terms of the expression

degree: the highest exponent of the variable in a term. The degree of $5x^4$ is 4; the degree of the constant term 5 is 0.

coefficient: in the term $2x$, the number 2 is a coefficient of the variable x

Step-By-Step

Example 6 asks you to subtract two expressions. To subtract an expression, change each of the terms to its opposite and then add. Here is a way to subtract vertically.

1 Line up the like terms.

$$\begin{array}{rrrr} 5a & & - & 11 \\ 12a & + b & - & 6 \end{array}$$

2 Change each term being subtracted to its opposite then add. Complete the addition.

$$\begin{array}{rrrr} 5a & & - & 11 \\ -12a & - b & + & 6 \\ \hline -7a & - b & & \end{array}$$

NO WORK, NO CREDIT

Computing with Expressions

7 Find the product.

$$-2a\,(-4a - 1)$$

Ⓐ $8a + 2$
Ⓑ $8a^2 + 2a$
Ⓒ $8a - 2$
Ⓓ $8a^2 - 2a$

Step-By-Step

Use the Distributive Property to do the multiplication shown in **example 7**. You will need to multiply two times.

1 Multiply $-2a$ times $-4a$. Use a^2 to show the product of $a \cdot a$.

$$-2a \cdot -4a = 8a^2$$

2 Multiply $-2a$ times -1.

$$-2a \cdot -1 = \quad$$

3 Put the two products together.

$$8a^2 + \quad$$

Solving Equations

8 Solve.

$$2(x - 5) = x(1 + 6)$$

Ⓐ $x = 2$
Ⓑ $x = -2$
Ⓒ $x = \frac{1}{2}$
Ⓓ $x = -\frac{1}{2}$

Remember

To keep an equation balanced, always perform the same operation on both sides of the equation. Use opposite operations.

- To undo addition, subtract.
- To undo subtraction, add.
- To undo multiplication, divide.
- To undo division, multiply.

Step-By-Step

You will need to simplify the equation in **example 8** before you can solve it.

1 Simplify both sides of the equation.

$$2(x - 5) = x(1 + 6)$$

$$2x - 10 = 7x$$

2 Subtract $2x$ from both sides.

$$2x - 10 - 2x = 7x - 2x$$

3 Divide both sides by 5.

$$-10 = 5x$$

$$\quad = x$$

GO ON

Linear and Proportional Relationships

3 Rachel and her family stayed in a hotel last night and she made several phone calls from the room. The hotel charges \$0.75 for each call plus \$0.20 per minute. Write an equation showing the cost c for m minutes.

Equation: ____________________

Is the equation linear or directly proportional?

Answer: ____________________

Step-By-Step

Use a simple problem to help you write the equation.

1. Ask yourself: How much would a 3-minute call cost?

   cost = \$0.75 + (3 × ☐)

2. Restate the equation using the variables c for cost and m for number of minutes.

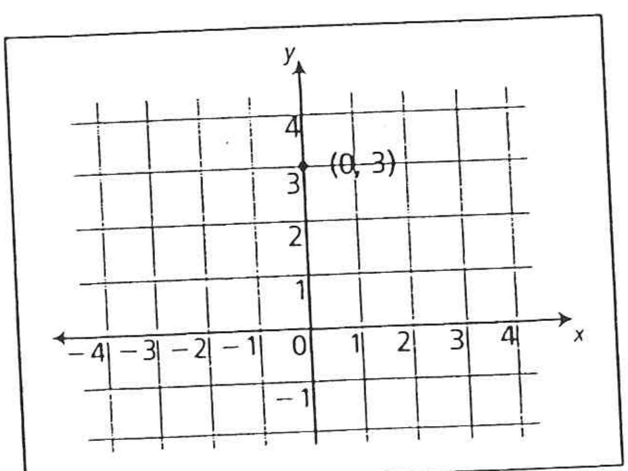
   $c =$ ☐

3. Refer to the chart on page 42 to determine if the equation is linear or a direct proportion.

Graphing Linear Situations

4 Complete the table of ordered pairs for the equation $y = 3 - \frac{1}{2}x$. Then use the table to draw a graph.

x	y
0	3
2	
4	



Step-By-Step

To solve example 4, substitute each given value of x into the equation. Then solve for y. This will give you three ordered pairs for the graph.

1. Find the values of y.

   $y = 3 - \frac{1}{2}(0)$ $y =$ 3

   $y = 3 - \frac{1}{2}(2)$ $y =$ ☐

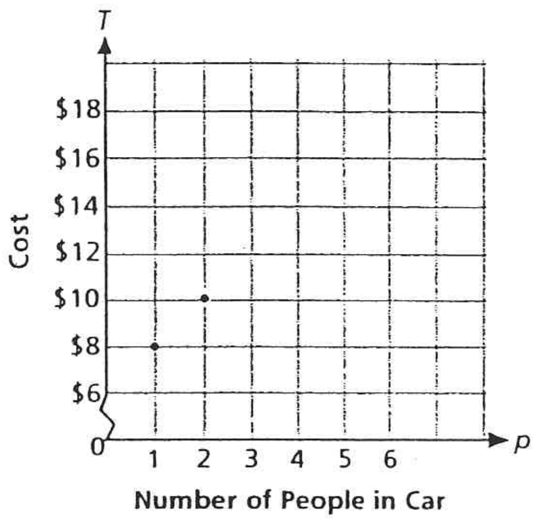
   $y = 3 - \frac{1}{2}(4)$ $y =$ ☐

2. Complete the table. Make a dot for each ordered pair. Then connect the dots with a straight line.

NO WORK, NO CREDIT

Graphing Linear Situations

5 Sunset Drive-In Movies is celebrating its anniversary and giving movie-goers a price discount. Instead of charging $6.50 per person, they are charging $6 per car plus $2 for each person in the car. Write an equation to find the admission cost for a car with from 1 to 6 people. Then plot the equation on the graph below.



Tip...

It does not make sense to draw a line through the points in example 5. The number of people cannot be a fraction.

Step-By-Step

1 Write an equation. T represents the total cost, and p represents the number of people in the car

$$T = \$6 + \$2p$$

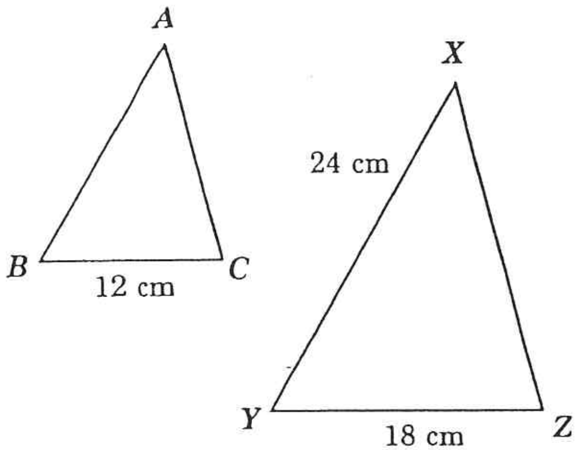
2 Make a table of ordered pairs and graph the points.

p	T
1	$8
2	$10
3	
4	
5	
6	

GO ON

Similar Polygons

Use this figure for examples 10 and 11. The two triangles shown are similar.



10 Which side completes the proportion?

$$\frac{AB}{XY} = \frac{?}{YZ}$$

Ⓐ AB Ⓒ AC
Ⓑ BC Ⓓ XZ

11 What is the measure of side AB?

Ⓐ 6 cm Ⓒ 16 cm
Ⓑ 8 cm Ⓓ 20 cm

Cross Multiplication

Use cross multiplication to eliminate fractions from a proportion. These two statements are equivalent.

$$\frac{a}{b} = \frac{c}{d} \qquad a \bullet d = b \bullet c$$

Step-By-Step

Examples 10 and 11 are based on similar triangles. Remember that corresponding sides of similar polygons are proportional. A proportion shows that two ratios are equal.

1 It may help you say the proportion out loud like this—

AB is to XY as ____ is to YZ.

2 Notice that the bottom elements of the ratios belong to triangle XYZ. This means that the top elements belong to triangle ABC.

3 Ask yourself: Which side of triangle ABC corresponds to side YZ?

Use the given lengths to set up a proportion to solve **example 11**. The missing term in the proportion will equal the length of side AB.

1 Write a proportion with x in the place of the unknown length. Be careful to set up the proportion so that corresponding sides fall in corresponding parts of the ratios. Which measure fits in the blank?

$$\frac{x}{24} = \frac{12}{\quad}$$

2 Cross-multiply.

$$24 \bullet 12 = \quad\quad \bullet x$$

3 Solve the proportion for x.

$$24 \bullet 12 = 18x$$
$$288 = 18x$$
$$288 \div 18 = x$$

$x =$ ____ cm

NO WORK, NO CREDIT