Mathematics Chart

**LENGTH**

Metric
- 1 kilometer = 1,000 meters
- 1 meter = 100 centimeters
- 1 centimeter = 10 millimeters

Customary
- 1 mile = 1,760 yards
- 1 mile = 5,280 feet
- 1 yard = 3 feet
- 1 foot = 12 inches

**CAPACITY AND VOLUME**

Metric
- 1 liter = 1,000 milliliters

Customary
- 1 gallon = 4 quarts
- 1 gallon = 128 ounces
- 1 quart = 2 pints
- 1 pint = 2 cups
- 1 cup = 8 ounces

**MASS AND WEIGHT**

Metric
- 1 metric ton = 1,000 kilograms
- 1 kilogram = 1,000 grams
- 1 gram = 1,000 milligrams

Customary
- 1 ton = 2,000 pounds
- 1 pound = 16 ounces

---

**Perimeter**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>( P = 4s )</td>
</tr>
<tr>
<td>rectangle</td>
<td>( P = 2l + 2w ) or ( P = 2(l + w) )</td>
</tr>
</tbody>
</table>

**Area**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>square</td>
<td>( A = s^2 )</td>
</tr>
<tr>
<td>rectangle</td>
<td>( A = lw ) or ( A = bh )</td>
</tr>
<tr>
<td>triangle</td>
<td>( A = \frac{1}{2} bh ) or ( A = \frac{bh}{2} )</td>
</tr>
</tbody>
</table>

\( b \) = the length of the base of rectangle or triangle.

**Surface Area**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>cube</td>
<td>( SA = 6s^2 )</td>
</tr>
<tr>
<td>rectangular prism</td>
<td>( SA = 2lw + 2lh + 2wh )</td>
</tr>
</tbody>
</table>

**Volume**

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>cube</td>
<td>( V = s^3 )</td>
</tr>
<tr>
<td>rectangular prism</td>
<td>( V = lwh )</td>
</tr>
</tbody>
</table>

**Slope**

<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} )</td>
</tr>
<tr>
<td>slope-intercept</td>
</tr>
</tbody>
</table>

when \( y = mx + b \),
\( m \) = slope and \( b \) = \( y \)-intercept.
Rate of Work

5. A person-hour is equivalent to one person doing 1 hour of work. If a job requires 20 person-hours, it could be accomplished by 20 people each working 1 hour, or 5 people each working 4 hours, or any combination with a product of 20. Suppose it takes 1,500 person-hours to complete a job. How many hours will it take 50 workers to do the job?

② 3 hours  ③ 25 hours  ④ 15 hours  ⑤ 30 hours

Converting Speeds

6. Karen’s stride is 18 inches. During her 15-minute walk to school, she counted 1,128 strides. What is Karen’s walking speed in feet per minute?

① 112.8 ft per min  ② 1.353.6 ft per min  ③ 1,692 ft per min  ④ 6,768 ft per min

Step-By-Step

Distance Measurement Facts

1 mi = 5,280 ft  1 km = 1,000 m
1 yd = 3 ft  1 m = 100 cm
1 ft = 12 in.  1 cm = 10 mm

For example 5, divide to find the number of hours each of 50 workers needs to work to complete the job.

1,500 person-hours ÷ 50 = hrs

Step-By-Step

To solve example 6, start by finding the distance in inches.

1. Multiply to find the total distance in inches Karen walked.

\[
\text{strides} \times \text{in./stride} = \text{total inches}
\]

\[
1,128 \times 18 = \text{in.}
\]

2. Divide by 12 to convert inches to feet.

\[
\text{inches} \div \text{inches/foot} = \text{total feet}
\]

\[
20.304 \div 12 = \text{ft}
\]

3. You know that Karen walked 1,692 feet in 15 minutes. Write a ratio and divide to find the unit rate.

\[
\frac{1.692 \text{ ft}}{15 \text{ min}} = \frac{?}{1 \text{ min}}
\]

\[
1,692 \div 15 = \text{ft per min}
\]
Solving Proportions

7 What is the missing term in this proportion?
\[ \frac{10}{15} = \frac{n}{9} \]
- @ 20
- @ 6
- @ 22.5
- @ 5

Remember

When reading a proportion such as the one above, it may help to convert it into a sentence.

10 is to 15 as \( n \) is to 9.

Scale Drawings

8 Georgia drew a scale drawing of a Model T car. She used a scale of 1 cm = 2 ft. What is the actual length of the Model T?

Key: 1 cm = 2 ft

Answer: ____________

Step-By-Step

There are several ways to solve example 7. One is shown below.

1 Use cross-multiplication to change the proportion to an equation
\[ 15 \cdot n_1 = 10 \cdot 9 \text{ or } 15n = 90 \]

2 To solve, divide both sides by 15. Complete the last step.
\[ 15n \div 15 = 90 \div 15 \]
\[ n = \]

Step-By-Step

You will need a centimeter ruler to solve example 8.

1 Measure the length of the car to the nearest centimeter. Use centimeters because the scale is given in centimeters.

\[ \text{length} = \]

2 Write a proportion.
\[ \frac{\text{scale length}}{\text{actual length}} = \frac{1 \text{ cm}}{2 \text{ ft}} = \frac{? \text{ cm}}{?} \]

3 Solve the proposition using cross products.
\[ \frac{1}{2} = \frac{?}{x} \]
\[ 1x = 2 \times 7 \]
\[ x = \]

GO ON
Constructing Responses Practice

Some tests include constructed-response questions in which you must show your work and explain your solution. The example below will give you practice responding to such questions.

9 A 6-foot person casts a shadow 4 feet long. The shadow of a flagpole is 8 feet long. How tall is the flagpole?

Step-by-Step

Example 9 involves using similar triangles and a proportion. Start by labeling the diagram at the left to show the three given terms of the proportion.

1 Write the proportion.

\[
\frac{\text{6-ft person}}{\text{4-ft shadow}} = \frac{n}{\text{8-ft shadow}}
\]

2 Cross-multiply.

\[4n = 6 \cdot 8 \]

3 Solve the equation by completing the last step.

\[4n = 48\]

\[n = 48 \div 4\]

4 Explain your steps using complete sentences.

The ratio of the person to his shadow is the same as the ratio of the flagpole to its shadow. Two similar triangles show this relationship. I used the triangle to write a proportion and then solved it for the missing term.
Test Practice 2:
Rates, Averages, and Proportions

Estimated time: 45 minutes

Directions: Read each question. Choose the best answer or write the answer to the question in the space you are given.

1. The coach wants to find the mean number of yards run. He has found the sum of the yards. What does he have to do next to find the mean?

<table>
<thead>
<tr>
<th>Yards Run</th>
<th>9</th>
<th>12</th>
<th>5</th>
<th>7</th>
<th>6</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>21</td>
<td>15</td>
<td>11</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

① Find the middle number.
② Divide by 2.
③ Subtract 21 from 12.
④ Divide by 12.

5. How far can you bicycle in 3 hours if you achieve an average speed of 15 mi/hr?
① 5 mi
② 18 mi
③ 30 mi
④ 45 mi

6. Each science project is given a score from 0 to 100 by each judge. The teacher's score counts twice as much as each of the three student judge's scores. Here are Marge's scores on her science project.

Student judges: 80, 75, 78
Teacher judge: 82

What is Marge's final score?
① 63
② 78.75
③ 79.4
④ 99.25

7. Suppose it takes 480 person-hours to complete a job. How many hours will 6 workers need to work to do the job?
① 8 hours
② 40 hours
③ 30 hours
④ 80 hours
⑤ 48 hours

8. Philip can run a mile in 6 minutes. What is his speed in miles per hour?
① 6 mi/hr
② 10 mi/hr
③ 12 mi/hr
④ 60 mi/hr
Computing with Rational Numbers

9 Find the quotient.
\[ \frac{8}{15} \div \left( -\frac{2}{5} \right) = \]
- \[ \frac{16}{75} \]
- \[ 1 \frac{1}{3} \]
- \[ -\frac{16}{75} \]
- \[ -1 \frac{1}{3} \]

Step-By-Step
In example 9 you are asked to divide rational numbers. Rational numbers are numbers that can be written as the ratio of two integers.

1 Divide as if the numbers were positive fractions.
\[ \frac{8}{15} \div \frac{2}{5} = \frac{8}{15} \times \frac{5}{2} = \]

2 Use the rules for dividing integers. The signs are different, so the quotient is negative.
\[ \frac{8}{15} \div \left( -\frac{2}{5} \right) = \]

3 Simplify by changing the fraction to a mixed number.
\[ -\frac{4}{3} = \]

Dividing by Fractions
To divide by a fraction, multiply by the reciprocal of the divisor.
- The reciprocal of \( \frac{3}{16} \) is \( \frac{16}{3} \).
- The reciprocal of 5 is \( \frac{1}{5} \).
- The reciprocal of \( 1 \frac{1}{4} \) is \( \frac{4}{5} \).

10 Find the sum.
\[ -\frac{1}{10} + \frac{5}{6} = \]
- \[ \frac{11}{15} \]
- \[ 1 \frac{1}{15} \]
- \[ -\frac{11}{16} \]
- \[ -\frac{14}{15} \]

Remember
When adding or subtracting fractions, you must first find a common denominator. Then add or subtract the numerators.

Step-By-Step
For example 10, begin by finding a common denominator.

1 What is the least common multiple of 10 and 6?
\[ \text{LCM}(10, 6) = \]

2 Use the LCM, 30, to write equivalent fractions.
\[ \frac{1}{10} \times \frac{3}{3} = \frac{3}{30} \quad \frac{5}{8} \times \frac{5}{5} = \frac{25}{30} \]

3 Add the numerators. Simplify.
\[ -\frac{3}{30} + \frac{25}{30} = \]
Estimating with Rational Numbers

11 Choose the best estimate for this quotient.

\[ 3.507 \div 0.028 = \]

\(\bigcirc 1.2 \quad \bigcirc 120 \quad \bigcirc 12 \quad \bigcirc 1,200 \)

Remember

The symbol \(=\) means "is approximately equal to."

12 Choose the best estimate for this sum.

\[ 4 \frac{5}{16} + 2 \frac{5}{8} = \]

\(\bigcirc \text{ less than } 6 \quad \bigcirc \text{ between } 6 \text{ and } 7 \quad \bigcirc \text{ between } 7 \text{ and } 8 \quad \bigcirc \text{ greater than } 8 \)

Remember

A fraction is greater than \(\frac{1}{2}\) if the numerator is greater than one half the denominator. These fractions are greater than \(\frac{1}{2}\): \[\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{8}{15}, \frac{13}{25}\]
Algebra: Expressions and Equations

Examples 1-13: Read each question. Choose the best answer or write the answer to the question in the space you are given.

Properties of Real Numbers

1. Which number sentence illustrates the Commutative Property?

- $7 + 6 = 8 + 5$
- $(4 + 8) + 10 = (5 + 7) + 10$
- $15 + (10 + 25) = 15 + (25 + 10)$
- $15 + (10 + 25) = (15 + 10) + 25$

Step-By-Step

For example 1, review the basic real number properties shown at the bottom of this page.

1. Read the definition for the Commutative Property of Addition. Which answer choices have the same numbers on each side of the equation?

2. Which answer changes the order of the numbers without changing the way they are grouped?

2. Fill the blank with a number that will show the Associative Property of Multiplication.

$18 \cdot (42 \cdot 36) = (18 \cdot 42) \cdot \_\_\_\_\_\_\_\_$

Step-By-Step

Review the Associative Property in the chart below. It states that you can group factors in different ways without changing the product.

<table>
<thead>
<tr>
<th>Commutative Property</th>
<th>Associative Property \hspace{1cm}</th>
<th>Distributive Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>The order in which you you add or multiply does not change the answer</td>
<td>You can group addends or factors in different ways</td>
<td>The product of a sum equals the sum of two products</td>
</tr>
<tr>
<td>$12 + 37 = 37 + 12$</td>
<td>$(14 + 7) + 6 = 14 + (7 + 6)$</td>
<td>$3(12 + 8) = (3 \times 12) + (3 \times 8)$</td>
</tr>
<tr>
<td>$2.1 \cdot 6 = 6 \cdot 2.1$</td>
<td>$(5 \cdot 2) \times 8 = 5 \times (2 \cdot 8)$</td>
<td>$(\frac{1}{2} \cdot 14) + (\frac{1}{2} \cdot 10) = \frac{1}{2}(14 + 10)$</td>
</tr>
</tbody>
</table>
Properties of Real Numbers

3 Use properties of operations and mental math to solve this problem.

\[ 25 \div (36 + 75) \times 1 = \]

Answer:________________

4 What is the reciprocal of 0.75?

\[ 1\frac{5}{7} \]  \( \bigcirc \) \[ 1\frac{4}{7} \]

\[ 1\frac{3}{3} \]  \( \bigcirc \) \[ 1\frac{12}{4} \]

Step-By-Step

Before solving example 3, review the four properties in the chart at the bottom of this page.

1 First use the Identity Property of Multiplication. Multiplying by 1 will not change the sum of \((36 + 75)\).

2 Use the Commutative and Associative Properties to simplify the addition.

\[ 25 \div (36 + 75) = (25 + 75) + 36 \]

3 Use mental math to add.

\[ (25 + 75) + 36 = \]

Step-By-Step

For example 4, apply the Inverse Property of Multiplication.

1 Write 0.75 as a fraction in lowest terms.

\[ 0.75 = \frac{75}{100} = \]

2 Interchange the numerator and denominator to find the reciprocal.

The reciprocal of \( \frac{3}{4} \) is \( \frac{4}{3} \)

3 Change the fraction to a mixed number.

\[ \frac{4}{3} = \]

<table>
<thead>
<tr>
<th>Property</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity Property of Addition</td>
<td>The sum of any number and 0 is the original number.</td>
<td>( x + 0 = x )</td>
</tr>
<tr>
<td>Identity Property of Multiplication</td>
<td>The product of any number and 1 is the original number.</td>
<td>( y \cdot 1 = y )</td>
</tr>
<tr>
<td>Inverse Property of Addition</td>
<td>The sum of any number and its opposite is 0.</td>
<td>( x + (-x) = 0 )</td>
</tr>
<tr>
<td>Inverse Property of Multiplication</td>
<td>The product of any number and its reciprocal is 1.</td>
<td>( y \cdot \frac{1}{y} = 1 )</td>
</tr>
</tbody>
</table>
Linear and Proportional Relationships

3 Rachel and her family stayed in a hotel last night and she made several phone calls from the room. The hotel charges $0.75 for each call plus $0.20 per minute. Write an equation showing the cost c for m minutes.

Equation: ____________________

Is the equation linear or directly proportional?

Answer: ____________________

Graphing Linear Situations

4 Complete the table of ordered pairs for the equation \( y = 3 - \frac{1}{2}x \). Then use the table to draw a graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Step-By-Step

To solve example 4, substitute each given value of \( x \) into the equation. Then solve for \( y \). This will give you three ordered pairs for the graph.

1 Find the values of \( y \).

\[
\begin{align*}
y &= 3 - \frac{1}{2}(0) \quad \Rightarrow \quad y = 3 \\
y &= 3 - \frac{1}{2}(2) \quad \Rightarrow \quad y = 2 \\
y &= 3 - \frac{1}{2}(4) \quad \Rightarrow \quad y = 1 \\
\end{align*}
\]

2 Complete the table. Make a dot for each ordered pair. Then connect the dots with a straight line.
Graphing Linear Situations

5 Sunset Drive-In Movies is celebrating its anniversary and giving movie-goers a price discount. Instead of charging $6.50 per person, they are charging $6 per car plus $2 for each person in the car. Write an equation to find the admission cost for a car with from 1 to 6 people. Then plot the equation on the graph below.

Step-By-Step

1 Write an equation. \( T \) represents the total cost, and \( p \) represents the number of people in the car

\[
T = 6 + 2p
\]

2 Make a table of ordered pairs and graph the points.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8</td>
</tr>
<tr>
<td>2</td>
<td>$10</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Tip...

It does not make sense to draw a line through the points in example 5. The number of people cannot be a fraction.
7 Connect dots to draw an isosceles triangle.

Questions 8–9: Use these similar figures to answer the questions.

8 Which angle corresponds to $\angle K$?
   - $\angle P$
   - $\angle Q$
   - $\angle R$
   - $\angle S$

9 What is the length of $SP$?
   - $1 \frac{1}{2}$ in.
   - $3$ in.
   - $2 \frac{1}{4}$ in.
   - $5 \frac{1}{3}$ in.

Questions 10–11: Use the rectangle below to draw the figures described.

10 Use a straightedge to draw a diagonal.

11 Use a compass to find the midpoint of side $FG$. Draw a point at the midpoint and label it $M$.

12 Which of these is a set of similar polygons?
   - all rectangles
   - all parallelograms
   - all equilateral triangles
   - all right triangles

13 On the grid below, draw rectangle $ABCD$ so that $AB$ is the width and $BC$ is the length. Then, draw triangle $ABE$ so that $ABE$ is isosceles.