Mathematics Chart

LENGTH

Metric	Customary
1 kilometer = 1,000 meters	1 mile = 1,760 yards
1 meter = 100 centimeters	1 mile = 5,280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

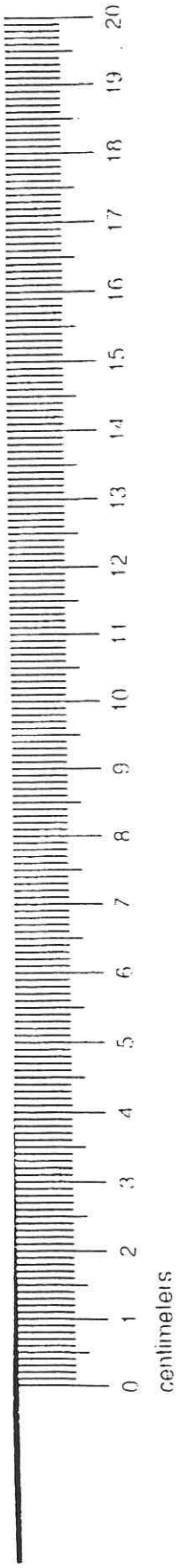
CAPACITY AND VOLUME

Metric	Customary
1 liter = 1,000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 metric ton = 1,000 kilograms	1 ton = 2,000 pounds
1 kilogram = 1,000 grams	1 pound = 16 ounces
1 gram = 1,000 milligrams	

Perimeter	square	$P = 4s$
	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Area	square	$A = s^2$
	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
b = the length of the base of rectangle or triangle.		
Surface Area	cube	$SA = 6s^2$
	rectangular prism	$SA = 2lw + 2lh + 2wh$
Volume	cube	$V = s^3$
	rectangular prism	$V = lwh$
Slope	slope formula	$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
	slope-intercept	when $y = mx + b$, m = slope and b = y-intercept



Rate of Work

5 A person-hour is equivalent to one person doing 1 hour of work. If a job requires 20 person-hours, it could be accomplished by 20 people each working 1 hour, or 5 people each working 4 hours, or any combination with a product of 20. Suppose it takes 1,500 person-hours to complete a job. How many hours will it take 50 workers to do the job?

Ⓐ 3 hours
Ⓑ 15 hours
Ⓒ 25 hours
Ⓓ 30 hours

Step-By-Step

For **example 5**, divide to find the number of hours each of 50 workers needs to work to complete the job.

1,500 person-hours ÷ 50 = _____ hrs

Converting Speeds

6 Karen's stride is 18 inches. During her 15-minute walk to school, she counted 1,128 strides. What is Karen's walking speed in feet per minute?

Ⓐ 112.8 ft per min
Ⓑ 1,353.6 ft per min
Ⓒ 1,692 ft per min
Ⓓ 6,768 ft per min

Distance Measurement Facts

1 mi = 5,280 ft	1 km = 1,000 m
1 yd = 3 ft	1 m = 100 cm
1 ft = 12 in.	1 cm = 10 mm

Step-By-Step

To solve **example 6**, start by finding the distance in inches.

1 Multiply to find the total distance in inches Karen walked.

strides × in./stride = total inches

1,128 × 18 = _____ in.

2 Divide by 12 to convert inches to feet.

inches ÷ inches/foot = total feet

20,304 ÷ 12 = _____ ft

3 You know that Karen walked 1,692 feet in 15 minutes. Write a ratio and divide to find the unit rate.

$$\frac{1{,}692 \text{ ft}}{15 \text{ min}} = \frac{?}{1 \text{ min}}$$

1,692 ÷ 15 = _____ ft per min

Solving Proportions

7 What is the missing term in this proportion?

$$\frac{10}{15} = \frac{n}{9}$$

Ⓐ 20 Ⓒ 6

Ⓑ 22.5 Ⓓ 5

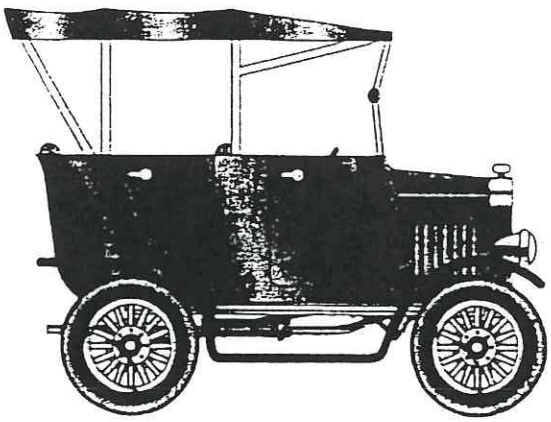
Remember

When reading a proportion such as the one above, it may help to convert it into a sentence.

10 is to 15 as n is to 9.

Scale Drawings

8 Georgia drew a scale drawing of a Model T car. She used a scale of 1 cm = 2 ft. What is the actual length of the Model T?



Key: 1 cm = 2 ft

Answer: _____

Step-By-Step

There are several ways to solve **example 7**. One is shown below.

1 Use cross-multiplication to change the proportion to an equation

$$15 \cdot n = 10 \cdot 9 \text{ or } 15n = 90$$

2 To solve, divide both sides by 15. Complete the last step.

$$15n \div 15 = 90 \div 15$$

$$n =$$

Step-By-Step

You will need a centimeter ruler to solve **example 8**.

1 Measure the length of the car to the nearest centimeter. Use centimeters because the scale is given in centimeters.

length = cm

2 Write a proportion.

$$\frac{\text{scale length}}{\text{actual length}} = \frac{1 \text{ cm}}{2 \text{ ft}} = \frac{7 \text{ cm}}{?}$$

3 Solve the proposition using cross products.

$$\frac{1}{2} = \frac{7}{x}$$

$$1x = 2 \times 7$$

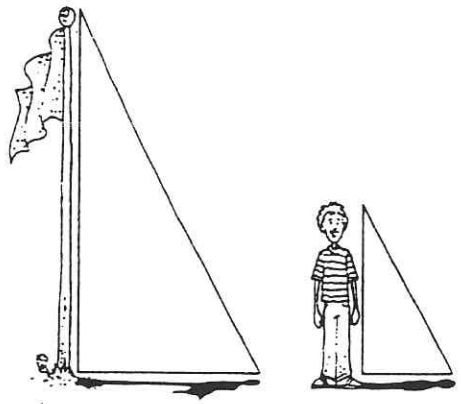
$$x =$$

GO ON

Constructed-Response Practice

Some tests include constructed-response questions in which you must show your work and explain your solution. The example below will give you practice responding to such questions.

9 A 6-foot person casts a shadow 4 feet long. The shadow of a flagpole is 8 feet long. How tall is the flagpole?



Writing Proportions

When writing a proportion such as the one in example 9, common elements need to be kept together.

Show all your work. Explain how you got your answer.

_____

_____

_____

_____

_____

Step-By-Step

Example 9 involves using similar triangles and a proportion. Start by labeling the diagram at the left to show the three given terms of the proportion.

1 Write the proportion.

$$\frac{\text{6-ft person}}{\text{4-ft shadow}} = \frac{n}{\text{8-ft shadow}}$$

2 Cross-multiply.

$$4n = 6 \cdot 8$$

3 Solve the equation by completing the last step.

$$4n = 48$$

$$4n \div 4 = 48 \div 4$$

$$n =$$

4 Explain your steps using complete sentences.

The ratio of the person to his shadow is the same as the ratio of the flagpole to its shadow. Two similar triangles show this relationship. I used the triangle to write a proportion and then solved it for the missing term.

Test Practice 1: Rates, Averages, and Proportions

Estimated time: 45 minutes

Directions: Read each question. Choose the best answer or write the answer to the question in the space you are given.

1 The coach wants to find the mean number of yards run. He has found the sum of the yards. What does he have to do next to find the mean?

Yards Run					
9	12	5	7	6	4
8	21	15	11	12	8

Ⓐ Find the middle number.
Ⓑ Divide by 2.
Ⓒ Subtract 21 from 12.
Ⓓ Divide by 12.

2 Solve for n.

$$\frac{9}{24} = \frac{n}{16}$$

Ⓐ 6 Ⓒ 19
Ⓑ 13.5 Ⓓ 42.7

3 The density of a diamond is 3.52 grams per cubic centimeter. Sally has a diamond that has a volume of 20 mm^3. Which shows the weight of the diamond?

Ⓐ 70.4 g Ⓒ 0.704 g
Ⓑ 7.04 g Ⓓ 0.0704 g

4 The ratio of fifth graders to sixth graders in a science club is 4 to 7. There are 20 fifth graders. How many sixth graders are there?

Ⓐ 11 Ⓒ 35
Ⓑ 28 Ⓓ 70

5 How far can you bicycle in 3 hours if you achieve an average speed of 15 mi/hr?

Ⓐ 5 mi Ⓒ 30 mi
Ⓑ 18 mi Ⓓ 45 mi

6 Each science project is given a score from 0 to 100 by each judge. The teacher's score counts twice as much as each of the three student judge's scores. Here are Marge's scores on her science project.

Student judges: 80, 75, 78
Teacher judge: 82

What is Marge's final score?

Ⓐ 63 Ⓒ 79.4
Ⓑ 78.75 Ⓓ 99.25

7 Suppose it takes 480 person-hours to complete a job. How many hours will 6 workers need to work to do the job?

Ⓐ 8 hours Ⓒ 80 hours
Ⓑ 40 hours Ⓓ 48 hours

8 Philip can run a mile in 6 minutes. What is his speed in miles per hour?

Ⓐ 6 mi/hr Ⓒ 12 mi/hr
Ⓑ 10 mi/hr Ⓓ 60 mi/hr

GO ON

Computing with Rational Numbers

9 Find the quotient.

$$\frac{8}{15} \div \left(-\frac{2}{5}\right) =$$

Ⓐ $\frac{16}{75}$ Ⓒ $1\frac{1}{3}$

Ⓑ $-\frac{16}{75}$ Ⓓ $-1\frac{1}{3}$

Dividing by Fractions

To divide by a fraction, multiply by the reciprocal of the divisor.

- The reciprocal of $\frac{3}{16}$ is $\frac{16}{3}$.
- The reciprocal of 5 is $\frac{1}{5}$.
- The reciprocal of $1\frac{1}{4}$ is $\frac{4}{5}$.

10 Find the sum.

$$-\frac{1}{10} + \frac{5}{6} =$$

Ⓐ $\frac{11}{15}$ Ⓒ $\frac{14}{15}$

Ⓑ $-\frac{11}{15}$ Ⓓ $-\frac{14}{15}$

Remember

When adding or subtracting fractions, you must first find a common denominator. Then add or subtract the numerators.

Step-By-Step

In **example 9** you are asked to divide rational numbers. **Rational numbers** are numbers that can be written as the ratio of two integers.

1 Divide as if the numbers were positive fractions.

$$\frac{8}{15} \div \frac{2}{5} = \frac{8}{15} \times \frac{5}{2} =$$

2 Use the rules for dividing integers. The signs are different, so the quotient is negative.

$$\frac{8}{15} \div \left(-\frac{2}{5}\right) =$$

3 Simplify by changing the fraction to a mixed number.

$$-\frac{4}{3} =$$

Step-By-Step

For **example 10**, begin by finding a common denominator.

1 What is the least common multiple of 10 and 6?

$$\text{LCM}(10, 6) =$$

2 Use the LCM, 30, to write equivalent fractions.

$$\frac{1}{10} \times \frac{3}{3} = \frac{3}{30} \qquad \frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$$

3 Add the numerators. Simplify.

$$-\frac{3}{30} + \frac{25}{30} = \qquad =$$

Estimating with Rational Numbers

11 Choose the best estimate for this quotient.

$$3.507 \div 0.028 \approx$$

Ⓐ 1.2
Ⓑ 12
Ⓒ 120
Ⓓ 1,200

Remember

The symbol ≈ means "is approximately equal to."

Step-By-Step

For **example 11**, use compatible numbers. Round the two numbers in the problem so that you can do the computation more easily.

1 Round the divisor first. Round to the nearest hundredth.

0.028 rounds to

2 Round the dividend to 3.6 so the computation will be easy.

$$3.6 \div 0.03 =$$

$$3.507 \div 0.028 \approx$$

12 Choose the best estimate for this sum.

$$4\tfrac{9}{16} + 2\tfrac{5}{8} \approx$$

Ⓐ less than 6
Ⓑ between 6 and 7
Ⓒ between 7 and 8
Ⓓ greater than 8

Remember

A fraction is greater than $\frac{1}{2}$ if the numerator is greater than one half the denominator. These fractions are greater than $\frac{1}{2}$.

$$\frac{3}{4} \quad \frac{5}{8} \quad \frac{7}{12} \quad \frac{8}{15} \quad \frac{13}{25}$$

Step-By-Step

For **example 12**, compare the fractional parts of the mixed numbers with $\frac{1}{2}$.

1 Both fractional parts are greater than $\frac{1}{2}$, so their sum is greater than 1.

2 Add the whole number parts. Then add 1 to that sum to estimate the sum of the fractional parts.

$$4\tfrac{9}{16} + 2\tfrac{5}{8} \approx$$

3 To figure out if the estimate is less than or greater than 7, think:

$$4\tfrac{1}{2} + 2\tfrac{1}{2} = 7$$

so—

$4\frac{9}{16} + 2\frac{5}{8}$ must be greater than 7.

GO ON

MATH

Lesson 3

Algebra: Expressions and Equations

Properties of Real Numbers
Computing with Expressions
Solving Equations
Applying Equations

Examples 1–13: Read each question. Choose the best answer or write the answer to the question in the space you are given.

Properties of Real Numbers

1 Which number sentence illustrates the Commutative Property?

Ⓐ $7 \div 6 = 8 \div 5$
Ⓑ $(4 \div 8) \div 10 = (5 \div 7) \div 10$
Ⓒ $15 \div (10 \div 25) = 15 \div (25 \div 10)$
Ⓓ $15 \div (10 \div 25) = (15 \div 10) \div 25$

Step-By-Step

For **example 1**, review the basic real number properties shown at the bottom of this page.

1 Read the definition for the Commutative Property of Addition. Which answer choices have the same numbers on each side of the equation?

2 Which answer changes the order of the numbers without changing the way they are grouped?

2 Fill the blank with a number that will show the Associative Property of Multiplication.

$18 \bullet (42 \bullet 36) = (18 \bullet 42) \bullet$ _____

Step-By-Step

Review the *Associative Property* in the chart below. It states that you can group factors in different ways without changing the product.

Commutative Property	Associative Property	Distributive Property
The order in which you **add** or **multiply** does not change the answer	You can group **addends** or **factors** in different ways	The product of a sum equals the sum of two products
$12 + 37 = 37 + 12$	$(14 + 7) + 6 = 14 + (7 + 6)$	$3(12 + 8) = (3 \times 12) + (3 \times 8)$
$2.1 \bullet 6 = 6 \bullet 2.1$	$(5 \times 2) \times 8 = 5 \times (2 \times 8)$	$(\frac{1}{2} \bullet 14) + (\frac{1}{2} \bullet 10) = \frac{1}{2}(14 + 10)$

Properties of Real Numbers

3 Use properties of operations and mental math to solve this problem.

$$25 \div (36 \div 75) \times 1 =$$

Answer:_____

4 What is the reciprocal of 0.75?

Ⓐ $1\frac{5}{7}$ Ⓒ $1\frac{1}{4}$

Ⓑ $1\frac{1}{3}$ Ⓓ $1\frac{3}{4}$

Opposites and Reciprocals

When you add $+5$ to -5, you get 0. The numbers $+5$ and -5 are opposites. **Opposites** are the same distance from 0 on a number line.

When you multiply $\frac{2}{3}$ times $\frac{3}{2}$, you get $\frac{6}{6}$, which equals 1. The numbers $\frac{2}{3}$ and $\frac{3}{2}$ are **reciprocals**.

Step-By-Step

Before solving **example 3**, review the four properties in the chart at the bottom of this page.

1 First use the Identity Property of Multiplication. Multiplying by 1 will not change the sum of (36 + 75).

2 Use the Commutative and Associative Properties to simplify the addition.

$$25 + (36 + 75) = (25 + 75) + 36$$

3 Use mental math to add.

$$(25 + 75) + 36 =$$

Step-By-Step

For **example 4**, apply the Inverse Property of Multiplication.

1 Write 0.75 as a fraction in lowest terms.

$$0.75 = \frac{75}{100} =$$

2 Interchange the numerator and denominator to find the reciprocal.

The reciprocal of $\frac{3}{4}$ is .

3 Change the fraction to a mixed number.

$$\frac{4}{3} =$$

Property	Definition	Example
Identity Property of Addition	The sum of any number and 0 is the original number.	$x + 0 = x$
Identity Property of Multiplication	The product of any number and 1 is the original number.	$y \cdot 1 = y$
Inverse Property of Addition	The sum of any number and its opposite is 0.	$x + (-x) = 0$
Inverse Property of Multiplication	The product of any number and its reciprocal is 1.	$y \cdot \frac{1}{y} = 1$

GO ON

Linear and Proportional Relationships

3 Rachel and her family stayed in a hotel last night and she made several phone calls from the room. The hotel charges \$0.75 for each call plus \$0.20 per minute. Write an equation showing the cost c for m minutes.

Equation: _____

Is the equation linear or directly proportional?

Answer: _____

Step-By-Step

Use a simple problem to help you write the equation.

1 Ask yourself: How much would a 3-minute call cost?

cost = \$0.75 + (3 ×)

2 Restate the equation using the variables c for cost and m for number of minutes.

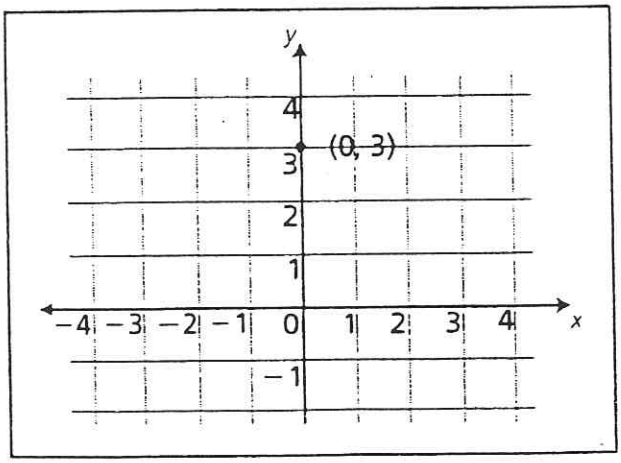
c =

3 Refer to the chart on page 42 to determine if the equation is linear or a direct proportion.

Graphing Linear Situations

4 Complete the table of ordered pairs for the equation $y = 3 - \frac{1}{2}x$. Then use the table to draw a graph.

x	y
0	3
2	
4	



Step-By-Step

To solve example 4, substitute each given value of x into the equation. Then solve for y. This will give you three ordered pairs for the graph.

1 Find the values of y.

$y = 3 - \frac{1}{2}(0)$ $y =$ 3

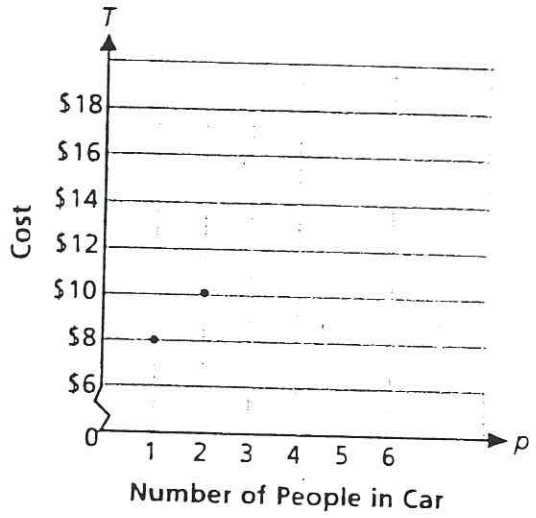
$y = 3 - \frac{1}{2}(2)$ $y =$

$y = 3 - \frac{1}{2}(4)$ $y =$

2 Complete the table. Make a dot for each ordered pair. Then connect the dots with a straight line.

Graphing Linear Situations

5 Sunset Drive-In Movies is celebrating its anniversary and giving movie-goers a price discount. Instead of charging \$6.50 per person, they are charging \$6 per car plus \$2 for each person in the car. Write an equation to find the admission cost for a car with from 1 to 6 people. Then plot the equation on the graph below.



Step-By-Step

1 Write an equation. T represents the total cost, and p represents the number of people in the car

$$T = \$6 + \$2p$$

2 Make a table of ordered pairs and graph the points.

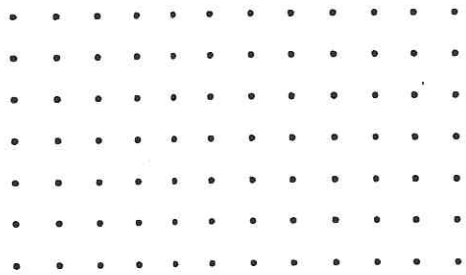
p	T
1	\$8
2	\$10
3	
4	
5	
6	

Tip...

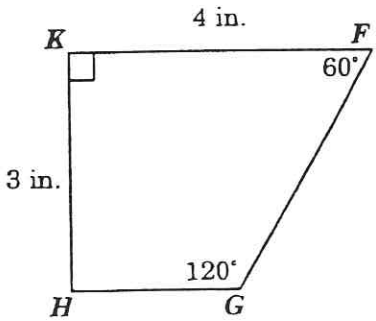
It does not make sense to draw a line through the points in example 5. The number of people cannot be a fraction.

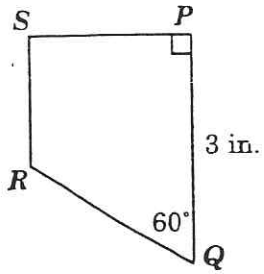
GO ON

7 Connect dots to draw an isosceles triangle.



Questions 8–9: Use these similar figures to answer the questions.





8 Which angle corresponds to $\angle K$?

Ⓐ $\angle P$ Ⓒ $\angle R$

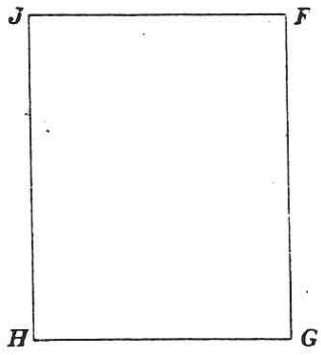
Ⓑ $\angle Q$ Ⓓ $\angle S$

9 What is the length of $\overline{SP}$?

Ⓐ $1\frac{1}{2}$ in. Ⓒ 3 in.

Ⓑ $2\frac{1}{4}$ in. Ⓓ $5\frac{1}{3}$ in.

Questions 10–11: Use the rectangle below to draw the figures described.



10 Use a straightedge to draw a diagonal.

11 Use a compass to find the midpoint of side $\overline{FG}$. Draw a point at the midpoint and label it M.

12 Which of these is a set of similar polygons?

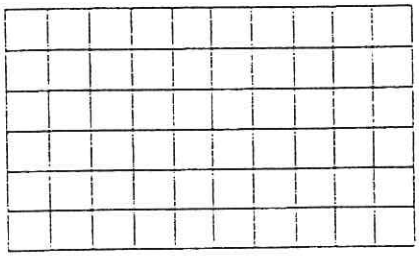
Ⓐ all rectangles

Ⓑ all parallelograms

Ⓒ all equilateral triangles

Ⓓ all right triangles

13 On the grid below, draw rectangle $ABCD$ so that AB is the width and BC is the length. Then, draw triangle ABE so that ABE is isosceles.



GO ON