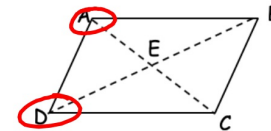


### Theorems to Prove Quadrilaterals are Parallelograms:

- ❖ If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- ❖ If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- ❖ If an angle of a quadrilateral is supplementary to both consecutive angles, then the quadrilateral is a parallelogram.
- ❖ If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- ❖ If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

3

Is there enough information to prove the quadrilateral is a parallelogram? If so, write a theorem as justification why it is a parallelogram.



1)  $\overline{AD} \parallel \overline{BC}$  and  $\overline{AD} \cong \overline{BC}$

yes

2)  $\overline{AE} \cong \overline{EC}$  and  $\overline{DE} \cong \overline{EB}$

yes

3)  $\overline{AB} \parallel \overline{CD}$  and  $\overline{AD} \cong \overline{BC}$

NO

4)  $\angle ADC \cong \angle CBA$  and  $\angle BAD \cong \angle DCB$

yes b/c both opp  $\angle$ s are  $\cong$ .

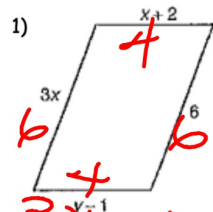
5)  $\angle DAB$  is supplementary to  $\angle ADC$  and  $\angle ABC$  is supplementary to  $\angle BCD$

NO

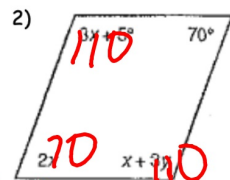
6)  $\triangle AED \cong \triangle CEB$

NO

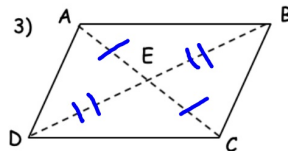
Determine the value of x and y so that the quadrilateral is a parallelogram.



$$\begin{aligned} 3x &= 6 \\ x &= 2 \\ 4 &= y - 1 \\ y &= 5 \end{aligned}$$



$$\begin{aligned} 2x &= 70 \\ x &= 35 \\ 110 &= 35 + 3y \\ 75 &= 3y \\ y &= 25 \end{aligned}$$



$$\begin{aligned} AE &= x^2 - 45, EC = -3x - 5 \\ DE &= 2y^2, EB = -3y + 2 \end{aligned}$$

$$\begin{aligned} x^2 - 45 &= -3x - 5 \\ x^2 + 3x - 40 &= 0 \\ x &= -8 \end{aligned}$$

$$2y^2 = -3y + 2$$

$$2y^2 + 3y - 2 = 0$$

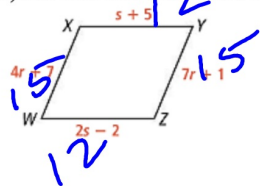
2y	2y <sup>2</sup>	4y
1	-1y	-2

$$(2y-1)(y+2) = 0$$

$$y = \frac{1}{2}, -2$$

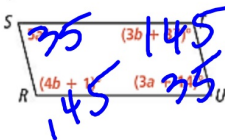
**Example 3:** Find values to make parallelogram.

A) For what values of  $r$  and  $s$  is WXYZ a parallelogram?



$r = 2$  yes this is a parallelogram  
 $s = 7$

B) For what values of  $a$  and  $b$  is RSTU a parallelogram? Which angle relationship is best to use and why?

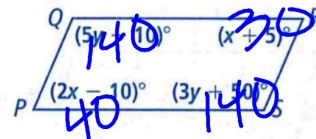


$a = 7$   $b = 36$   
 yes this is a parallelogram  
 opp  $\angle$ s are  $\cong$ .

Do Try It 3 and Habits of Mind, page 149 in your student companion.

**EXAMPLE 3 Try It! Find Values to Make Parallelograms**

3. a. If  $x = 25$  and  $y = 30$ , is PQRS a parallelogram?



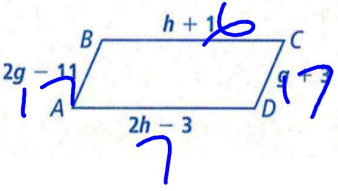
NO, this is not a parallelogram

**HABITS OF MIND**

**Reason** Given algebraic expressions for the angles of a quadrilateral, what properties could you use to decide if the quadrilateral is a parallelogram? © M22

If all variables in each exp. were the same, set the sum equal to 360

b. If  $g = 14$  and  $h = 5$ , is ABCD a parallelogram?



NO, this is not a parallelogram

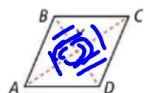
Theorem 6-14 (converse to 6-10):

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**Example 4:** Complete the proof of Theorem 6-14 below.

Given:  $\overline{AX} \cong \overline{CX}$  and  $\overline{BX} \cong \overline{DX}$

Prove:  $ABCD$  is a parallelogram



Statements	Reasons
1. $\overline{AX} \cong \overline{CX}$ and $\overline{BX} \cong \overline{DX}$	1. Given
2. $\angle AXD \cong \angle CXB$ and $\angle AXB \cong \angle CXD$	2. Vert $\angle$ s
3. $\triangle AXD \cong \triangle CXB$ and $\triangle AXB \cong \triangle CXD$	3. SAS
4. $\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$	4. CPCTC
5. $ABCD$ is a parallelogram.	5. Thm 6-11

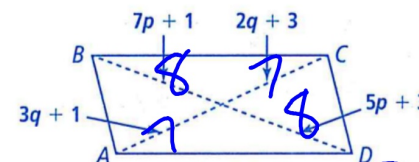
Do Try It 4, page 150 in your student companion.

Theorem 6-15:

If one pair of opposite sides of a quadrilateral is both  $\cong$  and  $\parallel$ , then the quadrilateral is a parallelogram.

**EXAMPLE 4** **Try It!** Investigate Diagonals to Confirm a Parallelogram

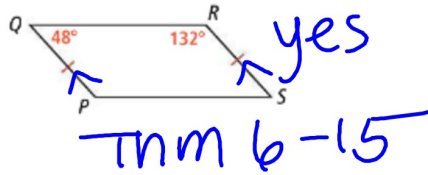
4. For what values of  $p$  and  $q$  is  $ABCD$  a parallelogram?



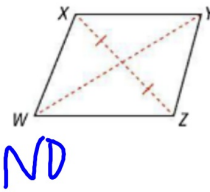
$$\begin{aligned}
 7p + 1 &= 5p + 3 & 3q + 1 &= 2q + 3 \\
 2p &= 2 & q &= 2 \\
 p &= 1 & &
 \end{aligned}$$

**Example 5: Are the following quadrilaterals also parallelograms? Explain.**

A)



B)



Do Try It 5 and Habits of Mind, page 150 in your student companion.

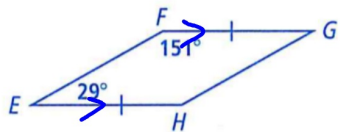
**EXAMPLE 5 Try It! Identify a Parallelogram**

5. a. Is ABCD a parallelogram? Explain.



NO, b/c  
only one pair  
of opp.  $\angle$ s  
 $\cong$

b. Is EFGH a parallelogram? Explain.



yes  
Thm 6-15

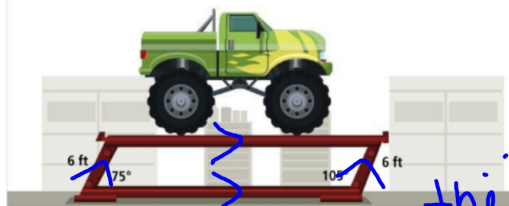
**HABITS OF MIND**

**Reason** Given algebraic expressions for the diagonals of a quadrilateral, what properties could you use to decide if the quadrilateral is a parallelogram? © MP.2

None, the diagonals  
bisect each other but  
are not necessarily  $\cong$ .



**Example 6:** A mechanic raises a truck using a lift. For safety, the floor must be horizontal, and the top of the lift must be parallel to the floor. Is the lift shown in a safe position? Explain.



Do Try It 6 and Habits of Mind, page 151 in your student companion

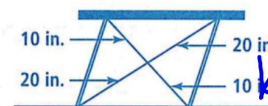
yes, b/c thm b-15  
this is a ll-gram  
the side of  
the lift holding the  
truck is ll  
to the ground

### HABITS OF MIND

**Make Sense and Persevere** How would you describe this problem in your own words? © MP.1

### EXAMPLE 6 Try It! Verify a Parallelogram

6. A carpenter builds the table shown. If the floor is level, how likely is it that a ball placed on the table will roll off?



b/c the diag.  
bisect each other  
this is a ll-gram  
∴ the table top  
is ll to the floor &  
the ball is not likely  
to roll off.

**In the book:** Read Concept Summary and #1-9, page 276 (page 152 in your student companion.)  
Tomorrow's HW: page 277 #10, 11, 13, 16-19, 21, 24, (3 possible points), 25, 26A, B