Sec 14-1: Trigonometric Identities

A trigonometric identity is an equation that is true for all values of x that are in the domain of the functions.

Said another way, a trigonometric identity is:

An equation in which both sides are ALWAYS equal

Trigonometric Expressions:

A trigonometric expression is an expression that contains trigonometric functions. Like all mathematical expressions, trigonometric expressions do not contain an equal sign (=).

Tools to use when simplifying Trigonometric Expressions:

Reciprocal identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

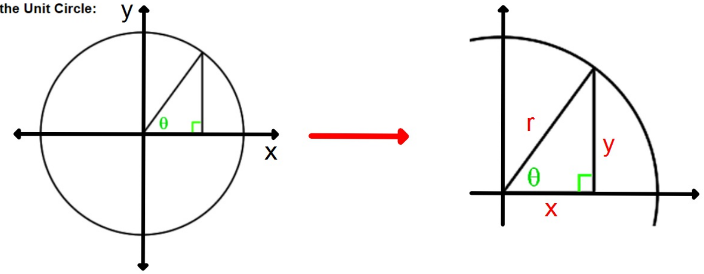
Tangent and cotangent identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

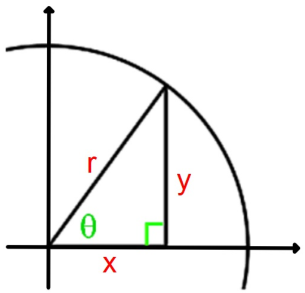
Another common tool that is used is called:

The Pythagorean Identity:

Let's start with the Unit Circle:



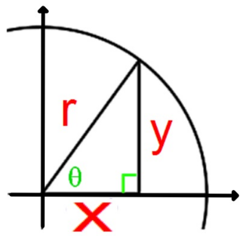
Using the Pythagorean Theorem we have:



$x^2 + y^2 = r^2$

$x^2 + y^2 = r^2$

$x = \cos\theta$
$y = \sin\theta$
$r = 1$



substituting for x, y, and r we get:

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

this is usually written as:

$\cos^2\theta + \sin^2\theta = 1$

or

$\sin^2\theta + \cos^2\theta = 1$

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

This is the Original Pythagorean Identity

Like a Parent Function, this leads to other

Pythagorean Identities:

rearranging this Pythagorean Identity leads to the following:

subtract $\cos^2\theta$ from sides by leads to the following identity:

$Sin^2\theta = 1 - Cos^2\theta$

subtract $\sin^2\theta$ from sides by leads to the following identity:

$Cos^2\theta = 1 - Sin^2\theta$

Pythagorean identities

The Original Pythagorean Identity: $\cos^2\theta + \sin^2\theta = 1$

this original Pythagorean Identity can be turned into two other ones:

divide both sides by $\cos^2\theta$ leads to the following identity:

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \qquad 1 + \tan^2\theta = \sec^2\theta$$

divide both sides by $\sin^2\theta$ leads to the following identity:

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \qquad 1 + \cot^2\theta = \csc^2\theta$$

Trigonometric Tools:

Basic Identities:

$\text{Tan}\theta = \dfrac{\text{Sin}\theta}{\text{Cos}\theta}$

$\text{Cot}\theta = \dfrac{1}{\text{Tan}\theta} = \dfrac{\text{Cos}\theta}{\text{Sin}\theta}$

$\text{Csc} = \dfrac{1}{\text{Sin}\theta}$

$\text{Sec} = \dfrac{1}{\text{Cos}\theta}$

Pythagorean Identities:

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

$\text{Sin}^2\theta = 1 - \text{Cos}^2\theta$

$\text{Cos}^2\theta = 1 - \text{Sin}^2\theta$

$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$

$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$

When you are simplifying a trigonometric expression you need to:

1. Know the rules.
2. Follow the rules.
3. Recognize that you can only multiply by 1.
4. Recognize that you can only add 0.

usually in the form of something over itself.

The rules come from definitions or identites that we have already proven.

Strategies for Simplifying Expressions

1) Change the expression into sines and cosines.
2) Look to use known formulas for purposes of substitution.
3) If there are fractions, gain a common denominator.
4) Use algebraic manipulations, like factoring, distributing, ...
5) If a strategy or substitution proves not to help, try something different.

this is probably the most important strategy. If you give up on your first attempt you will probably not be able to complete a lot of problems. It takes experience to get good at these problems and giving up won't make it any easier.

Simplify each trig expression:

1. sinx cotx

$= \cancel{\sin} \cdot \dfrac{\cos}{\cancel{\sin}}$

$= \boxed{\cos x}$

or

sinx cotx

$= \sin \cdot \dfrac{1}{\tan}$

$= \sin \cdot \dfrac{1}{\frac{\sin}{\cos}}$

$= \sin \cdot \dfrac{\cos}{\sin}$

$= \boxed{\cos x}$

2. $\dfrac{\sec x}{\csc x}$

$= \dfrac{\frac{1}{\cos}}{\frac{1}{\sin}}$

$= \dfrac{1}{\cos} \cdot \dfrac{\sin}{1}$

$= \dfrac{\sin}{\cos} = \boxed{\tan x}$

Simplify each trig expression:

3. cosx cscx

$= \cos x \cdot \frac{1}{\sin x}$

$= \frac{\cos x}{\sin x}$

$= \boxed{\cot x}$

4. $\frac{\text{cosx secx}}{\text{tanx}}$

$= \frac{\cos \cdot \frac{1}{\cos}}{\frac{\sin}{\cos}}$

$= \frac{1}{\frac{\sin}{\cos}}$

$= \frac{\cos}{\sin} = \boxed{\cot x}$

or

$\frac{\text{cosx secx}}{\text{tanx}}$

$\frac{\cos \cdot \frac{1}{\cos}}{\tan}$

$= \frac{1}{\tan}$

$= \boxed{\cot x}$