Starting points for y = aSin/Cosx

Sine Graphs

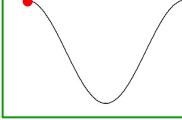**Positive a**

Starts on the Midline and goes UP

Negative a

Starts on the Midline and goes DOWN
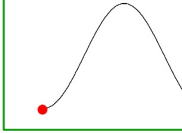
Cosine Graphs

Positive a

Starts at a Max

Negative a

Starts at a Min

$y = aSin(b(x \pm h)) \pm k$

a ⟶ Vert strech or shrink - Amplitude
    Also x-axis reflection if negative

b ⟶ Horiz stretch or shrink   Period = 2π/b and b = 2π/Period

h ⟶ Phase Shift - Horiz translation - the x-coord of start point

k ⟶ Vertical translation - Equation of the Midline

    gives you y-coord of pts on midline

$y = aCos(b(x \pm h)) \pm k$

a ⟶ Vert strech or shrink - Amplitude
    Also x-axis reflection if negative

b ⟶ Horiz stretch or shrink   Period = 2π/b and b = 2π/Period

h ⟶ Phase Shift - Horiz translation - x-coord of starting point
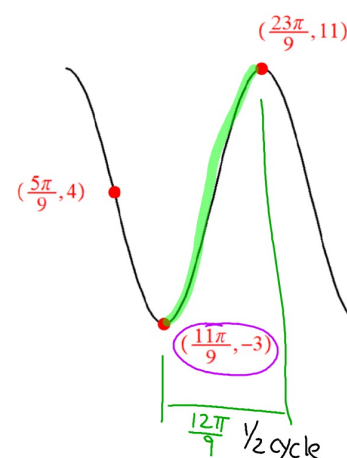
k ⟶ Vertical translation - Equation of the Midline
    gives you y-coord of pts on midline

This is ALL exactly the same as Sine. That's why if you know the Sine function well you already know most every thing you need to know for Cosine!

---

1. Write the equation of this transformed Cosine Function.

$(\frac{23\pi}{9}, 11)$

$(\frac{5\pi}{9}, 4)$

$(\frac{11\pi}{9}, -3)$

$\frac{12\pi}{9}$ ½ cycle

Amplitude: 11-4 = 7

midline: y = 4

$period = \dfrac{\frac{12\pi}{9}}{\frac{1}{2}} = \dfrac{12\pi}{9} \cdot \dfrac{2}{1} = \dfrac{8\pi}{3}$

• IF START AT $\left(\frac{11\pi}{9}, -3\right)$
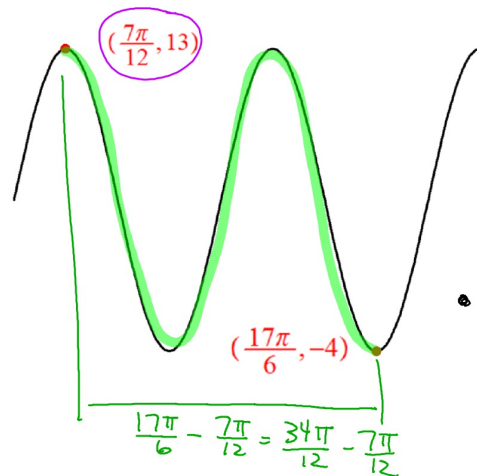
a = -7   (START AT A MIN Graph is upside down)

K = 4

$b = \dfrac{2\pi}{\frac{8\pi}{3}} = 2\pi \cdot \dfrac{3}{8\pi} = \dfrac{3}{4}$

phase shift: $\frac{11\pi}{9}$ TO THE RIGHT

$$y = -7 Cos\left(\frac{3}{4}\left(x - \frac{11\pi}{9}\right)\right) + 4$$

2. Write the equation of this transformed Cosine Function.

$\left(\frac{7\pi}{12}, 13\right)$

$\left(\frac{17\pi}{6}, -4\right)$

$\frac{17\pi}{6} - \frac{7\pi}{12} = \frac{34\pi}{12} - \frac{7\pi}{12}$

$= \frac{27\pi}{12} = \frac{9\pi}{4}$

$1\frac{1}{2} \text{ cycles} \Rightarrow \frac{3}{2}$

$Amp = \frac{13 - -4}{2} = \frac{17}{2}$

$midline: y = \frac{13 + -4}{2} = \frac{9}{2}$

$period = \frac{\frac{9\pi}{4}}{\frac{3}{2}} = \frac{9\pi}{4} \cdot \frac{2}{3}$

$= \frac{3\pi}{2}$

• If start at $\left(\frac{7\pi}{12}, 13\right)$

$a = +\frac{17}{2}$ (START AT MAX, not upside down)

$K = \frac{9}{2}$

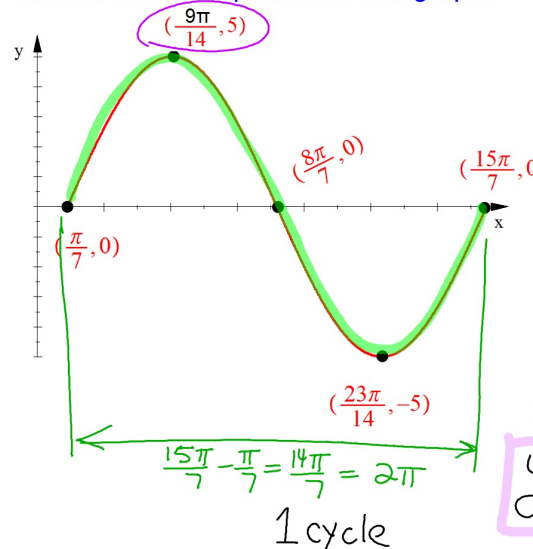$b = \frac{2\pi}{3\pi/2} = 2\pi \cdot \frac{2}{3\pi} = \frac{4}{3}$

phase shift: $\frac{7\pi}{12}$ RIGHT

$$y = \frac{17}{2} \cos\left(\frac{4}{3}\left(x - \frac{7\pi}{12}\right)\right) + \frac{9}{2}$$

Write a Cosine Equation for this graph.

$\left(\frac{9\pi}{14}, 5\right)$

$\left(\frac{8\pi}{7}, 0\right)$

$\left(\frac{15\pi}{7}, 0\right)$

$\left(\frac{\pi}{7}, 0\right)$

$\left(\frac{23\pi}{14}, -5\right)$

$\frac{15\pi}{7} - \frac{\pi}{7} = \frac{14\pi}{7} = 2\pi$

1 cycle

$Amp = 5 - 0 = 5$

$midline: y = 0$

$period = \frac{2\pi}{1} = 2\pi$

• If start at $\left(\frac{9\pi}{14}, 5\right)$

$a = +5$ (START AT MAX NOT upside down)
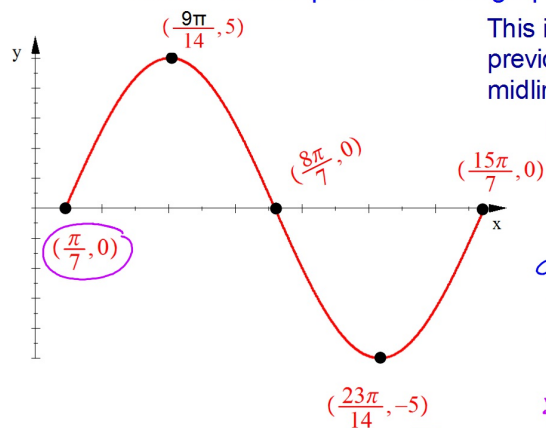
$K = 0$

$b = \frac{2\pi}{2\pi} = 1$

phase shift: $\frac{9\pi}{14}$ RIGHT

$$y = 5\cos\left(x - \frac{9\pi}{14}\right)$$

9π

Write a Sine Equation for this graph.

$\left(\frac{9\pi}{14}, 5\right)$

$\left(\frac{8\pi}{7}, 0\right)$

$\left(\frac{15\pi}{7}, 0\right)$

$\left(\frac{\pi}{7}, 0\right)$

$\left(\frac{23\pi}{14}, -5\right)$

This is the same graph as the previous question. The amplitude, midline, and period are all the same.

But the starting point will be different.
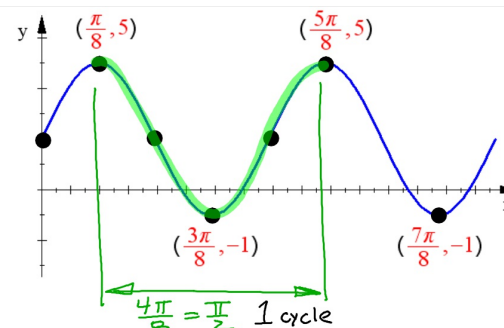
IF START AT $\left(\frac{\pi}{7}, 0\right)$

$a = +5$ (STARTS AT midline & goes up)

$K = 0$

$b = 1$

phase shift: $\frac{\pi}{7}$ Right

$$y = 5 \sin\left(x - \frac{\pi}{7}\right)$$

(bottom right panel)

$\left(\frac{\pi}{8}, 5\right)$

$\left(\frac{5\pi}{8}, 5\right)$

$\left(\frac{3\pi}{8}, -1\right)$

$\left(\frac{7\pi}{8}, -1\right)$

$\frac{4\pi}{8} = \frac{\pi}{2}$ 1 cycle

$Amp = \frac{5 - -1}{2} = 3$

$midline: y = \frac{5 + -1}{2} = 2$

$period = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$

Write the equation of this graph as a Cosine Function:

START AT $\left(\frac{\pi}{8}, 5\right)$ phase shift: $\frac{\pi}{8}$ Right

$a = +3$ (START AT MAX NOT upside down)

$K = 2$ $b = \frac{2\pi}{\pi/2} = 2\pi \cdot \frac{2}{\pi} = 4$

$$y = 3\cos\left(4\left(x - \frac{\pi}{8}\right)\right) + 2$$

Write the equation of this graph as a Sine Function:

Same period, amplitude, and midline as Cosine.

If start at point on y-axis $x = 0$, therefore, NO phase shift

$a = +3$ (START at midline & goes up)

$K = 2$ $b = 4$

$$y = 3 \sin 4x + 2$$

Graph one period of this function. Label the coordinates of
all x-intercepts, Maximums, and Minimums.

$y = -8\text{Cos}(\frac{1}{2}(x - \frac{5\pi}{6})) + 1$

Amp = 8
upside down

midline: $y = 1$

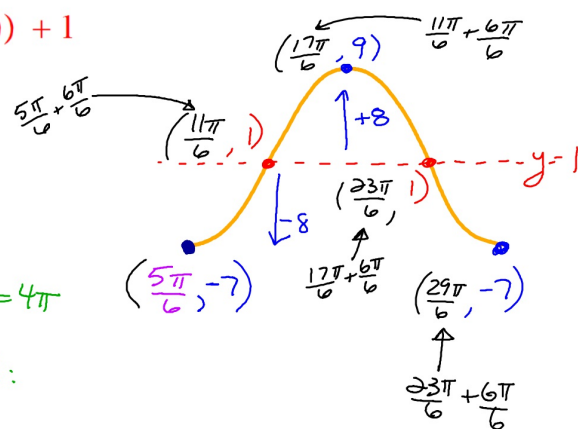phase shift: $\frac{5\pi}{6}$ Right

period = $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

one fourth of a period:

$\frac{1}{4} \cdot 4\pi = \pi$

to find the x-coord of any point add
π to the previous x-coord. To do
this you must turn π into 6π/6 to get
the common denominator.

$\frac{5\pi}{6} + \frac{6\pi}{6}$ $\frac{11\pi}{6} + \frac{6\pi}{6}$

$(\frac{17\pi}{6}, 9)$

$(\frac{11\pi}{6}, 1)$ $+8$

$(\frac{23\pi}{6}, 1)$ $y = 1$

-8

$(\frac{5\pi}{6}, -7)$ $\frac{17\pi}{6} + \frac{6\pi}{6}$ $(\frac{29\pi}{6}, -7)$

$\frac{23\pi}{6} + \frac{6\pi}{6}$

You can now finish Hwk #21.

Practice Sheet Sec 13-7 (Cosine)