

1. The number of cells of a certain bacteria doubles (increases by 100%) every 30 minutes. There are 500 cells at 2pm. Find the number of cells at each time.

a. Noon the same day

$$x = \frac{400 \text{ am} - 2 \text{ pm}}{30} = -6$$

$$y = 500(2)^{-6} = 419, 430, 400$$

b. 6:45am the same day

$$x = \frac{6:45 \text{ am} - 2 \text{ pm}}{30} = -9.5$$

$$y = 500(2)^{-9.5} = 289, 631$$

c. 10:15pm the previous day

$$x = \frac{10:15 \text{ pm} - 2 \text{ pm}}{30} = -7.5$$

$$y = 500(2)^{-7.5} = 2$$

2. Every 40 minutes the amount of medicine in your body decreases 50% (called half life). If you took 400mg dose at 10:00am find the amount of medicine in your system at each time.

a. 2:00pm the same day

$$y = 400(.5)^x$$

$$x = \frac{2:00 \text{ pm} - 10:00 \text{ am}}{40} = 6$$

$$y = 400(.5)^6 = 6.25 \text{ mg}$$

b. 4:30pm the same day

$$x = \frac{4:30 \text{ pm} - 10:00 \text{ am}}{40} = 7.5$$

$$y = 400(.5)^{7.5} = 0.46 \text{ mg}$$

3. Every five years the population of a city increases 11%. The population of this city in 1995 was 89,000. Find the population of this city in the following years to the nearest whole number.

a. 2005

$$y = 89000(1.11)^x$$

$$y = 89000(1.11)^2 = 109,657$$

b. 1982

$$x = \frac{-13}{5} = -2.6$$

67,850

c. 2017

$$x = \frac{22}{5} = 4.4$$

140,868

## Graphs of Exponential Functions

## General Form of an Exponential Equation:

$$y = a \cdot b^x$$

Exponent  
any real #

Base  
 $b > 0, b \neq 1$

Coefficient  
 $a \neq 0$

Graphs of  $y = a \cdot b^x$

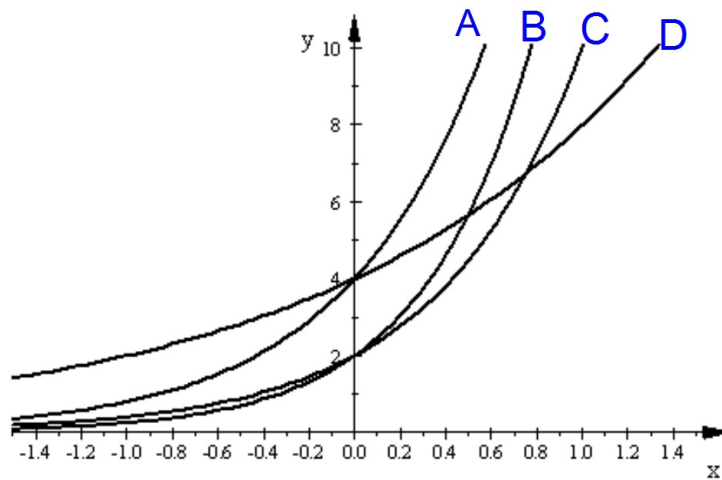
**a:** the y-intercept. If  $a$  is negative graph is upside down  
(x-axis reflection)

**b:** Growth or Decay Factor

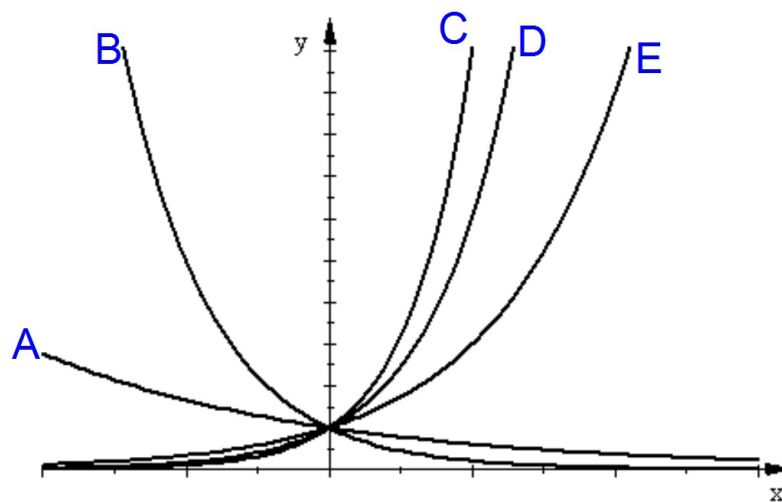
**Growth Factor:** The larger the value of  $b$  the faster the graph increases.  $b > 1$

**Decay Factor:** The smaller the value of  $b$  the faster the graph decreases  
 $0 < b < 1$

1 D  $y = 4(2)^x$    2 C  $y = 2(5)^x$    3 B  $y = 2(8)^x$    4 A  $y = 4(5)^x$



1 D  $y = 6^x$    2 B  $y = 0.5^x$    3 A  $y = 0.8^x$    4 C  $y = 10^x$



5 F  $y = 2^x$

Find the value of  $x$  in each equation:  
Round to the nearest hundredth when needed.

1.  $12x = 600$  50
2.  $64 = x^3$  4
3.  $10^5 = x$  100,000
4.  $10^x = 200$

Every math operation has its inverse.

Inverse operations "undo" each other.

We solve equations by using inverses to get the variable by itself.

Given Operation	Inverse Operation
Addition	Subtraction
Division	Multiplication
Squaring	Square Root
Cube Root	Cubing

Find the equation of the inverse for this function:

$$\begin{aligned}
 & y = \sqrt{\frac{4x^3 - 7}{8}} + 1 \quad \textcircled{1} \\
 & x = \sqrt[2]{\frac{4y^3 - 7}{8}} + 1 \quad \textcircled{2} \\
 & \sqrt[3]{\frac{8(x-1)^2 + 7}{4}} = y \quad \textcircled{3}
 \end{aligned}$$

Find the equation of the inverse.

$$y = 10^x$$

To solve for  $x$  in an exponential equation:  $y = 10^x$   
we use the inverse operation called:

**Logarithm**

Sec 8-3: Logarithms  
(the inverse of exponential functions)

Exponential Function

$$y = b^x$$

The base  
of the  
Exponential  
Function

The exponent

Logarithmic Function

$$\log_b y = x$$

The base of the  
Logarithmic  
Function

Exponential Function:

$$y = b^x$$

Logarithmic Function:

How do you say this?

$$\log_b y = x$$

The base is  
the base

The exponent is the answer



Another way to remember Logarithmic Form:

Exponential  
Form:

$$x = y^z$$

becomes

Logarithmic  
Form:

$$z = \text{Log}_y x$$

Exponential Equation

Range:  
 $y > 0$

Domain:  
Any real number

$$y = b^x$$

$b > 0, b \neq 1$

Logarithmic Equation

$$\log_b y = x$$

Range:  
Any real number

Domain:  
 $x > 0$

$b:$   $b > 0, b \neq 1$

Rewrite each into logarithmic form.

1.  $5^x = 40$        $\log_5 40 = x$

2.  $6^2 = x$        $\log_6 x = 2$

3.  $x^2 = 20$        $\log_x 20 = 2$

Rewrite each into exponential form.

1.  $\text{LOG}_5 8 = x$        $5^x = 8$

2.  $\text{LOG}_3 x = 12$        $3^{12} = x$

3.  $\text{LOG}_x 15 = 30$        $x^{30} = 15$

Write in Logarithmic Form:

$$10^x = 125$$

$$\text{LOG}_{10}125 \rightarrow \text{"LOG base 10 of 125"} \rightarrow \text{LOG}125$$

$\text{LOG}_{10}$  is called the Common Logarithm and is written without the 10.

The button on the calculator LOG is for Common Logarithms  $\text{LOG}_{10}$

Evaluate each: (hint: think of each as an exponential)

1.  $\log_4 1$   $0$   
 $4^x = 1$

2.  $\log_3 9$

$$3^x = 9 \quad x = 2$$

3.  $\log_7(7)$   $1$   
 $7^x = 7$   
 $x = 1$

4.  $\log_{25} 5$

$$25^x = 5$$
$$x = .5$$

5.  $\log_6(6^4)$

$$6^x = 6^4$$
$$x = 4$$

6.  $\log_2(0.5)$

$$2^x = 0.5$$
$$x = 1/2$$

7.  $\log 54$

$$1.73$$

Solve each equation. Round to the nearest tenth.

1.  $10^x = 1500$

$$\log_{10}(1500) = x$$
$$3.2$$

2.  $\frac{4(10)^x}{4} = \frac{570}{4}$

$$10^x = 142.5$$
$$x = 2.2$$

3.  $4^x = 44$

$$\log_4 44 = x$$

$$x = 2.7$$

$$\frac{\log_{10} 44}{\log_{10} 4}$$

4.  $12^x = 3$

$$\log_{12} 3 = x$$
$$x = 0.4$$

What if your calculator only has LOG?

**Property**

**Change of Base Formula**

For any positive numbers,  $M$ ,  $b$ , and  $c$ , with  $b \neq 1$  and  $c \neq 1$ ,

$$\log_b M = \frac{\log_c M}{\log_c b}$$

The value of a house has been decreasing 7.5% each year. The house was worth \$180,000 in 2001.

In how many years will the value fall to \$45,000? Round to the nearest hundredth.

$$\frac{45,000}{180,000} = \frac{180,000(0.925)^x}{180,000}$$
$$0.25 = 0.925^x$$
$$\log 0.925^x$$
$$0.25 = x$$
$$x = 18$$