

Completing the Square & Converting between Vertex & Standard Form

Rey

Quiz Practice:

1.) Find the value of n such that each expression is a perfect-square trinomial.

a.) $v^2 + 24v + n$

$$\frac{b}{2} = \frac{24}{2} = 12^2 = 144$$

$$n = 144$$

b.) $x^2 + 3x + n$

$$\frac{b}{2} = \frac{3}{2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

2.) Solve each quadratic by Completing the Square:

a.) $\frac{2k^2 + 4k = 10}{2}$

$$k^2 + 2k = 5$$

$$k^2 + 2k + \underline{1} = 5 + \underline{1}$$

$$(k+1)^2 = 6 \rightarrow k = -1 \pm \sqrt{6}$$

b.) $\frac{3k^2 + 12k = 24}{3}$

$$k^2 + 4k = 8$$

$$k^2 + 4k + \underline{4} = 8 + \underline{4}$$

$$(k+2)^2 = 12$$

$$k = -2 \pm \sqrt{12}$$

$y = ax^2 + bx + c$ - this is called: Standard form

What does a tell us about the parabola?

opens up/down
vertical stretch/shrink

What does b tell us about the parabola?

horizontal translation

What does c tell us about the parabola?

y intercept
vertical translation

$y = a(x-h)^2 + k$ - this is called: Vertex form

a:

h:

k:

Converting from Standard Form to Vertex Form:

Standard Form:

$$y = ax^2 + bx + c$$

We know: a, b, c

Vertex Form:

$$y = a(x - h)^2 + k$$

want a, h, k

$$a = a$$

$$x = -b/2a = h$$

$$\text{Solve for } y = k$$

Example 1:

$$y = 8x^2 - 16x + 27$$

We know a, b, c and want a, h, k

$$a = 8$$

← a is the coefficient of the x^2 term

$$h = x = \frac{-b}{2a} = \frac{16}{16} = 1$$

← use the formula to find the value of h

$$k = y = \frac{8(1)^2 - 16(1) + 27}{19}$$

← substitute the value found for h into the original equation and solve for k

$$y = 8(x - 1)^2 + 19$$

Example 2:

$$y = 5x^2 - 40x + 67$$

$$a = 5$$

$$h = x = \frac{-b}{2a} = \frac{40}{10} = 4$$

$$k = y = -13$$

$$y = 5(x - 4)^2 - 13$$

Quick Check 1:

$$y = 5x^2 - 10x + 37$$

$$a = 5$$

$$h = x = \frac{10}{2(5)} = 1$$

$$k = y = 5(1)^2 - 10(1) + 37 = 32$$

$$y = 5(x - 1)^2 + 32$$

Quick Check 2:

$$y = 7x^2 + 28x + 19$$

$$a = 7$$

$$h = x = \frac{-28}{14} = -2$$

$$k = y = 7(-2)^2 + 28(-2) + 19 = -9 \quad y = 7(x+2)^2 - 9$$

Quick Check 3:

$$y = -2x^2 - 24x - 75$$

$$a = -2$$

$$h = x = \frac{24}{2(-2)} = -6$$

$$y = -2(x+6)^2 - 3$$

$$k = y = -2(-6)^2 - 24(-6) - 75 = -3$$

Convert from Vertex Form to Standard Form:

$$y = a(x - h)^2 + k \quad \Rightarrow \quad y = ax^2 + bx + c$$

Example 1:

$$y = 5(x + 2)^2 - 9$$

$$5(x+2)(x+2) - 9 \quad \leftarrow \text{Rewrite } (x+2)^2$$

$$5(x^2 + 4x + 4) - 9 \quad \leftarrow \text{Simplify } (x+2)(x+2)$$

$$5x^2 + 20x + 20 - 9 \quad \leftarrow \text{Distribute the 5}$$

$$y = 5x^2 + 20x + 11 \quad \leftarrow \text{Combine Like Terms}$$

Example 2:

$$y = -3(x - 4)^2 + 7$$

$$\begin{aligned} y &= -3(x-4)(x-4) + 7 && \longleftarrow \text{Rewrite } (x-4)^2 \\ &= -3(x^2 - 8x + 16) + 7 && \longleftarrow \text{Simplify } (x-4)(x-4) \\ &= -3x^2 + 24x - 48 + 7 && \longleftarrow \text{Distribute the } -3 \\ &= -3x^2 + 24x - 41 && \longleftarrow \text{Combine Like Terms} \end{aligned}$$

Quick Check 1:

$$y = (x - 2)^2 + 6$$

$$\begin{aligned} y &= (x-2)(x-2) + 6 \\ &= x^2 - 4x + 4 + 6 \\ &\Rightarrow y = x^2 - 4x + 10 \end{aligned}$$

Quick Check 2:

$$y = 3(x - 3)^2 - 12$$

$$\begin{aligned} y &= 3(x-3)(x-3) - 12 \\ &= 3(x^2 - 6x + 9) - 12 \\ &= 3x^2 - 18x + 27 - 12 \\ &= 3x^2 - 12x + 15 \end{aligned}$$