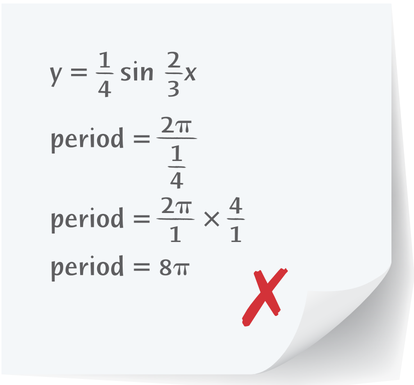
UNDERSTAND

8. Use Structure Write the equations of three cosine functions that have an amplitude of $\frac{1}{2}$ and that have periods of $\frac{1}{2}$, 2, and 4. Then graph and label all three equations on the same coordinate plane.

9. Look for Relationships Explain why the sine function is a periodic function.

10. Error Analysis Describe and correct the error a student made in solving for the period of the given function.



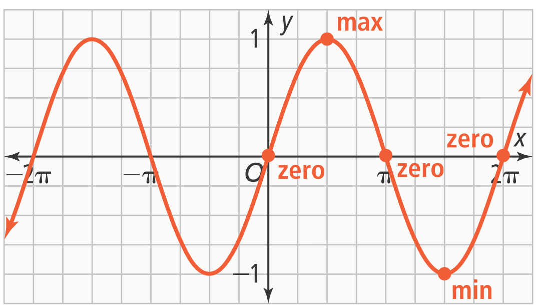
$y = \frac{1}{4} \sin \frac{2}{3}x$

$\text{period} = \frac{2\pi}{\frac{1}{4}}$

$\text{period} = \frac{2\pi}{1} \times \frac{4}{1}$

$\text{period} = 8\pi$

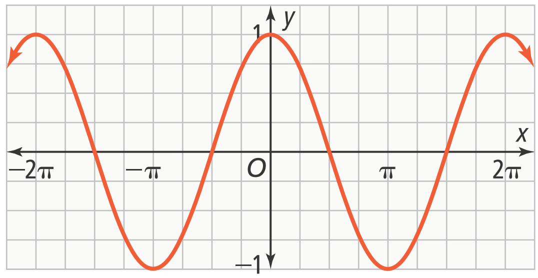
11. Look for Relationships A "five-point pattern" can be used to graph sine and cosine functions. The five-point pattern for the sine function when $a > 0$ is zero-max-zero-min-zero, as shown on the graph. What is the five-point pattern for the sine function when $a < 0$?



12. Higher Order Thinking Use a graphing calculator to graph $y = \sin x$ and $y = \csc x$. What do you notice about the graph of $y = \csc x$ where $y = 0$ on the graph of $y = \sin x$? (*Hint*: $y = \csc x$ is equivalent to $y = \frac{1}{\sin x}$.)

PRACTICE

13. Identify the domain, range, and period of the function $y = \cos x$. SEE EXAMPLE 1



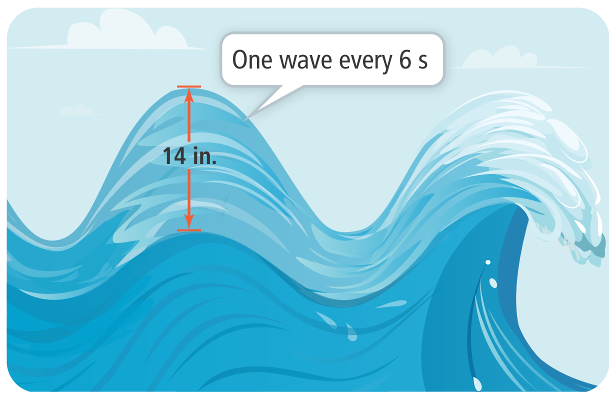
What are the amplitude and period of each function? SEE EXAMPLE 2

14. $y = \frac{1}{2} \cos \frac{1}{8}x$

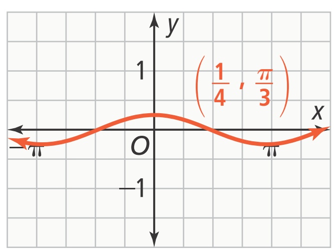
15. $y = 5 \sin \frac{1}{4}x$

16. Use technology to graph $y = \frac{3}{4} \sin 2x$. What is the frequency? What is the average rate of change on the interval $[0, \pi]$? SEE EXAMPLE 3

17. A particle in the ocean moves with a wave. The motion of the particle can be modeled by the cosine function. If a 14 in. wave occurs every 6 s, write a function that models the height of the particle in inches y as it moves in seconds x. What is the period of the function? SEE EXAMPLE 4



18. How to the periods of the two functions compare? SEE EXAMPLE 5



$f(x) = \frac{1}{4} \cos \frac{\pi}{3}x$

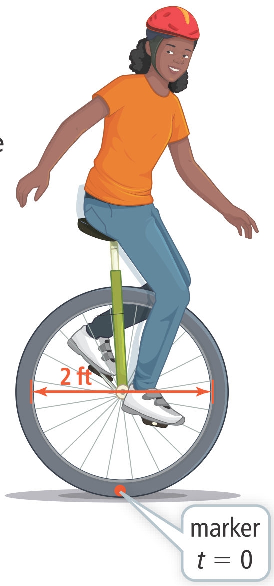
PRACTICE & PROBLEM SOLVING

Practice Tutorial

Mixed Review Available Online

APPLY

19. Make Sense and Persevere The relationship between the height of a point on a unicyle wheel, in feet, and time, in seconds, can be modeled by the sine function. A unicycle wheel has a diameter of 2 ft. A marker was placed on the wheel at time $t = 0$ s with a height of $h = 0$ ft. When Esteban is riding the unicycle, it takes $\frac{\pi}{2}$ s for the unicycle wheel to make one complete revolution.



a. What is the period of the function?

b. What is the amplitude of the function?

c. Write an equation to represent this situation.

d. Graph the function.

e. How many revolutions will the unicycle wheel make in 4π s when Esteban is riding the unicycle?

20. Model With Mathematics A solar day is 24 h and a lunar day is 24 h 50 min. A lunar day is 50 min longer than a solar day because the moon revolves around Earth, and Earth rotates around its axis in the same direction. This means it takes Earth 50 min longer to catch up with the moon. Each lunar day, two high tides and two low tides occur. High tides occur 12 h 25 min apart. Yesterday, high tide was measured at 8 ft above sea level and low tide was measured at 2 ft above sea level. A cosine function models the depth of the water in feet, D, at time t in hours.

a. What is the period of the function?

b. The amplitude is the difference between the depth of the water at high tide and the average depth of the water. What is the amplitude?

c. Write an equation to represent D as a function of t.

ASSESSMENT PRACTICE

21. Find the key features of the function $y = 8\cos\left(\frac{\pi}{6}x\right)$. Write the correct value from the box next to each key feature.

amplitude =

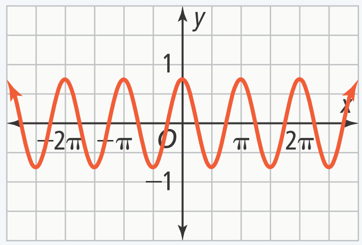
period =

frequency =

midline =

3	8	12
$\frac{1}{8}$	$\frac{1}{12}$	$\frac{\pi}{3}$
$x = 0$		$y = 0$

22. SAT/ACT What is the equation of the graph?



Ⓐ $y = \frac{3}{4}\cos(2x)$

Ⓑ $y = \frac{3}{2}\cos x$

Ⓒ $y = \frac{3}{4}\sin(2x)$

Ⓓ $y = \frac{3}{2}\sin x$

23. Performance Task Danielle is investigating how the signs of the parameters a and b create transformations of the sine function.

Part A Graph $y = (\sin 2x)$ and $y = -\sin(2x)$ on the same coordinate plane.

Part B How are the graphs of $y = \sin(2x)$ and $y = -\sin(2x)$ related?

Part C Graph $y = \sin(2x)$ and $y = \sin(-2x)$ on the same coordinate plane.

Part D How are the graphs of $y = \sin 2x$ and $y = \sin(-2x)$ related?

Part E How is the graph of $y = a\sin(bx)$ affected when a or b is replaced with its opposite? Explain.