

# Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2a}$

Polynomial Equations Day 3 Notes

Notable Student:

Key

Date:

Solve  $3x^2 + 4x + 4 = 0$ .

$a = 3$

$b = 4$

$c = 4$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(4)}}{2(3)} = \frac{-4 \pm \sqrt{16 - 48}}{6} = \frac{-4 \pm \sqrt{-32}}{6}$$

Put  $\sqrt{-32}$  into calc.

**2 problems:** we need to know what it means when we take the square root of a negative number and we need to know how to simplify square roots.

**Simplifying Square Roots:** Look at the table below and try to fill it out using your calculators to find the decimal approximations.

Radical Expression	Decimal Approximation	Radical Expression	Decimal Approximation
$\sqrt{40}$		$\sqrt{4} \cdot \sqrt{10}$	
$\sqrt{90}$		$\sqrt{9} \cdot \sqrt{10}$	
$\sqrt{98}$		$\sqrt{49} \cdot \sqrt{2}$	
$\sqrt{125}$		$\sqrt{25} \cdot \sqrt{5}$	

What do we notice? What does that tell us about square roots?

**Our best friends (perfect squares):**

1, 4, 9, 16, 25, 36, 49, 64, 81, 100  
 $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2$

**Practice:** Simplify the following:

1.  $\sqrt{128}$

2.  $\sqrt{150}$

3.  $\sqrt{196}$

4.  $\sqrt{97}$

$\sqrt{128} = \sqrt{64 \cdot 2} = 8\sqrt{2}$

$\sqrt{196} = \sqrt{49 \cdot 4} = 7 \cdot 2 = 14$

$\sqrt{150} = \sqrt{25 \cdot 6} = 5\sqrt{6}$

$\sqrt{97}$  (cannot be simplified)

**Negative Square Roots:** What does your calculator say when you enter  $\sqrt{-1}$ ?

Non-real calculation (or) error

Carl Friedrich Gauss discovered a new set of numbers, the **complex numbers** when trying to figure out the answer to  $\sqrt{-1}$ . Today we define this as "i" and we say  $\sqrt{-1} = i$

We can use this to simplify these radicals:

1.  $\sqrt{-36}$

$$\begin{aligned} &\sqrt{-1 \cdot 36} \\ &\rightarrow i \cdot 6 \\ &\text{(or)} \\ &6i \end{aligned}$$

2.  $\sqrt{-96}$

3.  $\sqrt{-245}$

$$\begin{aligned} &\sqrt{-1 \cdot 245} \\ &i \cdot \sqrt{49 \cdot 5} \end{aligned}$$

$$(i \cdot 7\sqrt{5}) \text{ or } (7i\sqrt{5})$$

Going back to our original problem: Solve  $3x^2 + 4x + 4 = 0$ .

$$\begin{aligned} X &= \frac{-4 \pm \sqrt{-32}}{6} = \frac{-4 \pm 4i\sqrt{2}}{6} = \frac{-4}{6} \pm \frac{4i\sqrt{2}}{6} = \frac{-2}{3} \pm \frac{2i\sqrt{2}}{3} \\ &\quad \downarrow \\ &\quad \sqrt{-32} \\ &\quad \downarrow \\ &\quad i\sqrt{16 \cdot 2} \\ &\quad \downarrow \\ &\quad 4i\sqrt{2} \end{aligned}$$

$$\frac{-2}{3} + \frac{2i\sqrt{2}}{3} \text{ (and)} \frac{-2}{3} - \frac{2i\sqrt{2}}{3}$$

**Practice:** Solve the following:

1.  $5x^2 - 2x + 5 = 0$

2.  $x^2 - 6x + 12 = 0$

3.  $9x^2 - 4x + 2 = 0$