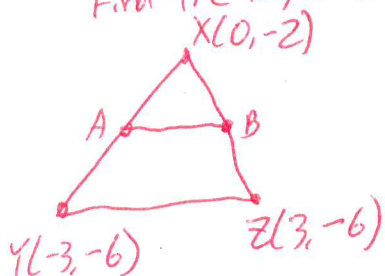


1. A and B are the midpoint of XY and XZ.
Find the length of AB.



Step 1: Find A

Step 2: Find B

Step 3: Find AB

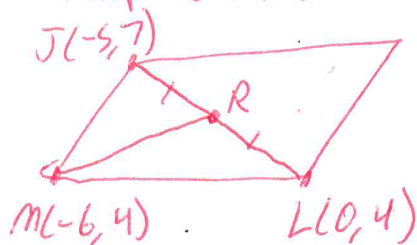
Step 2: B is the midpoint of XZ

$$\begin{array}{cc} X(0, -2) & Z(3, -6) \\ x_1 & x_2 \end{array}$$

$$\begin{aligned} x_m &= \frac{x_1 + x_2}{2} & y_m &= \frac{y_1 + y_2}{2} \\ x_m &= \frac{0 + 3}{2} & y_m &= \frac{-2 + -6}{2} \\ x_m &= \frac{3}{2} & y_m &= \frac{-8}{2} \\ & & y_m &= -4 \end{aligned}$$

$$B\left(\frac{3}{2}, -4\right)$$

2. Find the length of MR if R is the midpoint of JL.



Step 1: Find R

Step 2: Find JL

Step 1: J(-5, 7) L(0, 4)

$$x_m = \frac{x_1 + x_2}{2} \quad y_m = \frac{y_1 + y_2}{2}$$

$$x_m = \frac{-5 + 0}{2} \quad y_m = \frac{7 + 4}{2}$$

$$x_m = -\frac{5}{2} \quad y_m = \frac{11}{2}$$

$$R\left(-\frac{5}{2}, \frac{11}{2}\right)$$

Step 1: A is the midpoint of XY

$$\begin{array}{cc} X(0, -2) & Y(-3, -6) \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

$$x_m = \frac{x_1 + x_2}{2} \quad y_m = \frac{y_1 + y_2}{2}$$

$$x_m = \frac{0 + -3}{2} \quad y_m = \frac{-2 + -6}{2}$$

$$x_m = -\frac{3}{2} \quad y_m = \frac{-8}{2}$$

$$A\left(-\frac{3}{2}, -4\right) \quad y_m = -4$$

Step 3: Find AB

$$\begin{array}{cc} A\left(-\frac{3}{2}, -4\right) & B\left(\frac{3}{2}, -4\right) \\ x_1 & x_2 \end{array}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{\left(\frac{3}{2} - -\frac{3}{2}\right)^2 + (-4 - -4)^2}$$

$$d = \sqrt{\left(\frac{6}{2}\right)^2 + (0)^2}$$

$$d = \sqrt{\frac{36}{4}} = \sqrt{9} = 3$$

$$\boxed{AB = 3}$$

Step 2: R(-5/2, 11/2) M(-6, 4)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{\left(-\frac{5}{2} - -6\right)^2 + \left(\frac{11}{2} - 4\right)^2}$$

$$d = \sqrt{\left(-\frac{5}{2} + \frac{12}{2}\right)^2 + \left(\frac{11}{2} - \frac{8}{2}\right)^2}$$

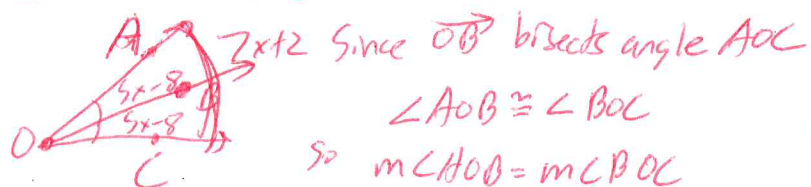
$$d = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$$

$$d = \sqrt{\frac{49}{4} + \frac{9}{4}}$$

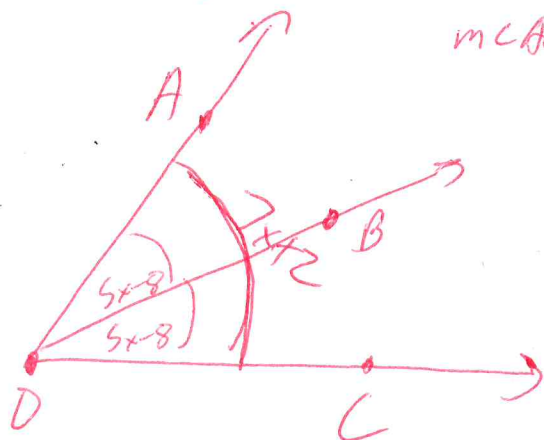
$$d = \sqrt{\frac{58}{4}} = \frac{\sqrt{58}}{2}$$

3. In the given figure, ray $\vec{OB}$ bisects $\angle AOC$. The measure of $\angle AOC$ is $7x+2$ and $m\angle COB = 5x-8$

What is the measure of $\angle AOB$?



Option 1



$$m\angle AOB + m\angle BOC = m\angle AOC$$

$$5x-8 + 5x-8 = 7x+2$$

$$10x-16 = 7x+2$$

$$-7x \quad -7x$$

$$3x-16 = 2$$

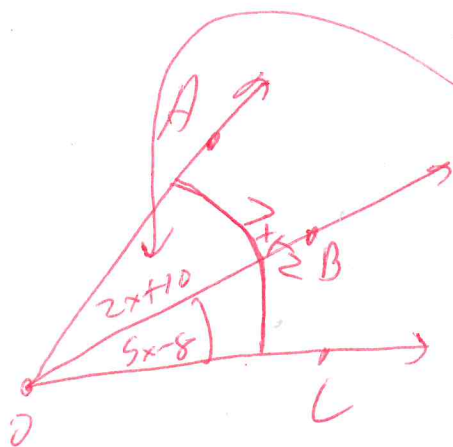
$$+16 \quad +16$$

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

$$\begin{aligned} m\angle AOB &= 5x-8 \\ &= 5(6)-8 \\ &= 30-8 \\ &= 22 \end{aligned}$$

Option 2



$$m\angle AOB + m\angle BOC = m\angle AOC$$

$$m\angle AOB + 5x-8 = 7x+2$$

$$-5x+8 \quad -5x+8$$

$$m\angle AOB = 2x+10$$

Since $\vec{OB}$ bisects $\angle AOC$, ~~$m\angle AOB = m\angle BOC$~~
 $\angle AOB \cong \angle BOC$

$$\text{So } m\angle AOB = m\angle BOC$$

$$2x+10 = 5x-8$$

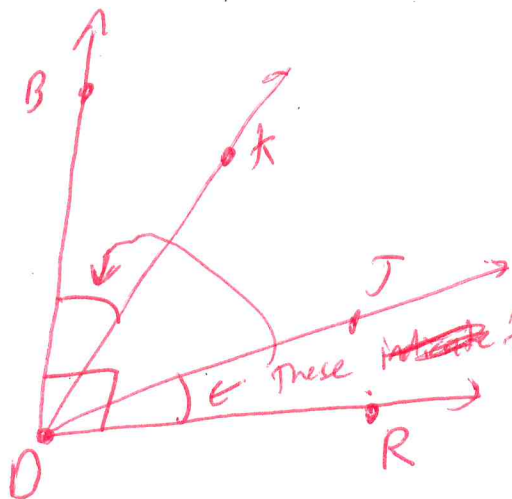
$$10 = 3x-8$$

$$18 = 3x$$

$$6 = x$$

$$\begin{aligned} m\angle AOB &= 2x+10 \\ &= 2(6)+10 \\ &= 12+10 \\ &= 22 \end{aligned}$$

4.

Find x

$$m\angle BDK = 3x + 4 \quad m\angle JDR = 5x - 10$$

These angles tell us that $\angle BDK \cong \angle JDR$

$$\text{so } m\angle BDK = m\angle JDR$$

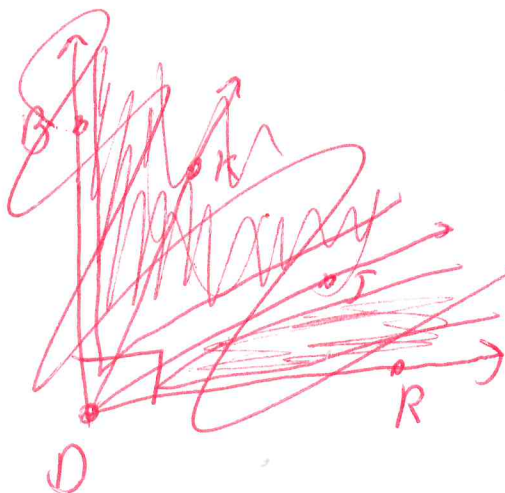
$$\text{so } 3x + 4 = 5x - 10$$

$$\begin{array}{r} -3x \quad -3x \\ 4 = 2x - 10 \\ +10 \quad +10 \end{array}$$

$$\frac{14}{2} = \frac{2x}{2}$$

$$\boxed{7 = x}$$

5.

Find y

$$m\angle BDJ = 7y + 2, \quad m\angle JDR = 2y + 7$$

We see that $m\angle BDR = 90$ since $\angle BDR$ is a right angle

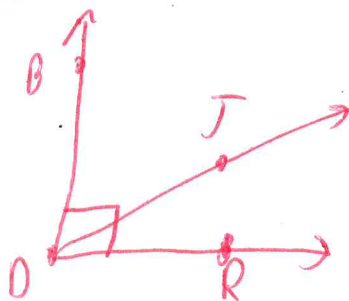
$$m\angle BDJ + m\angle JDR = m\angle BDR$$

$$7y + 2 + 2y + 7 = 90$$

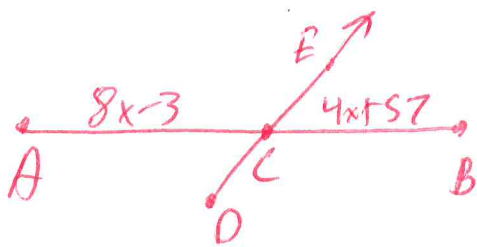
$$9y + 9 = 90$$

$$\frac{9y}{9} = \frac{81}{9}$$

$$\boxed{y = 9}$$



6. $\overrightarrow{DE}$ bisects $\overline{AB}$ at C . If $AC = 8x - 3$ and $CB = 4x + 57$, find AC .



Since $\overrightarrow{DE}$ bisects $\angle C$, $\overline{AC} \cong \overline{CB}$

So $AC = CB$

$$\text{so } \begin{array}{r} 8x - 3 = 4x + 57 \\ -4x \quad -4x \end{array}$$

$$\begin{array}{r} 4x - 3 = 57 \\ +3 \quad +3 \end{array}$$

$$\frac{4x}{4} = \frac{60}{4}$$

$1x = 15$

7-9) Q is in the interior of $\angle ROS$, S is in the interior of $\angle QOP$, P is in the interior of $\angle SOT$. $m\angle ROT = 127^\circ$
 $m\angle SOT = 71^\circ$

and $m \in R \overset{x}{O} Q \neq m \in Q \overset{x}{O} S = m \in P \overset{x}{O} T$

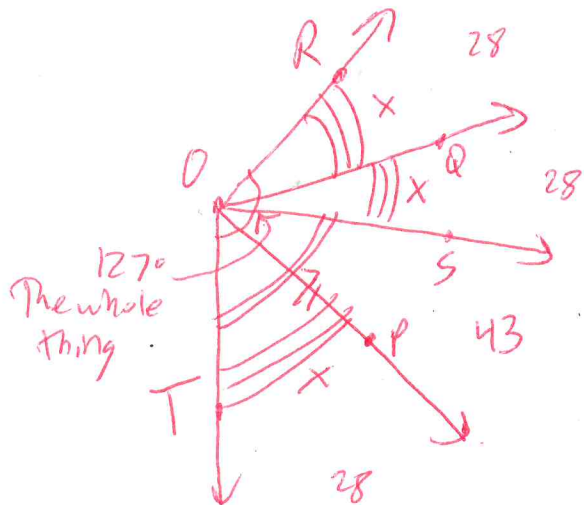
* Let x equal all three of these angles

$$x + x + 71 = 127$$

$$2x + 71 = 127$$

$$\frac{2x}{2} = \frac{56}{2}$$

$$X = 28$$



$$m\angle 50^\circ = 71 - 28$$
$$m\angle 50^\circ = 43$$

7) Find MCQOP

$$m\angle QOS + m\angle SOP = m\angle QOP$$

$$28 + 43 = m \angle QOP$$

$71 = m \angle QOP$

8) Find $m\angle QOT$

$$m\angle QOP + m\angle POT = m\angle QOT$$

$$71 + 28 = m\angle QOT$$

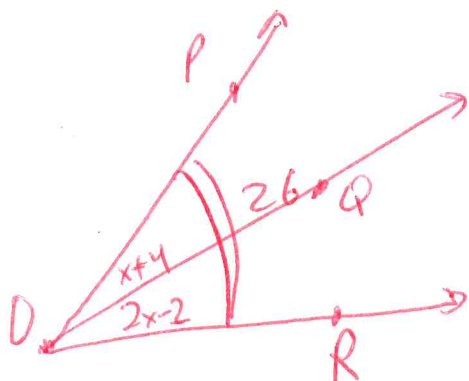
$$99 = m \angle QOT$$

9) Find $m \in \mathbb{R}$

$$x = m \in \mathbb{R} \cap \mathbb{Q}$$

$$28 = m \angle ROQ$$

10. Let Q be in the interior of $\angle POR$. Solve for x. Find measure of each angle.



$$\begin{aligned} m\angle POQ &= x+4 \\ m\angle QOR &= 2x-2 \\ m\angle POR &= 26 \end{aligned}$$

$$m\angle POQ + m\angle QOR = m\angle POR$$

$$x+4 + 2x-2 = 26$$

$$3x+2 = 26$$

$$\begin{array}{r} -2 \quad -2 \\ 3x = 24 \\ \hline x = 8 \end{array}$$

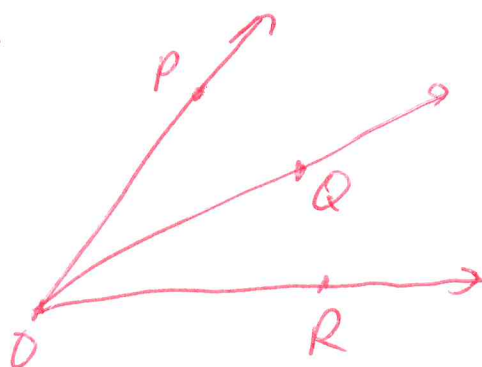
$$\boxed{x=8}$$

Find measure

$$\begin{aligned} m\angle POQ &= x+4 \\ m\angle POQ &= 8+4 \\ m\angle POQ &= 12 \end{aligned}$$

$$\begin{aligned} m\angle QOR &= 2x-2 \\ m\angle QOR &= 2(8)-2 \\ m\angle QOR &= 16-2 \\ m\angle QOR &= 14 \end{aligned}$$

11.



$$\begin{aligned} m\angle POQ &= 3x+7 \\ m\angle QOR &= 5x-2 \\ m\angle POR &= 61 \end{aligned}$$

$$m\angle POQ + m\angle QOR = m\angle POR$$

$$3x+7 + 5x-2 = 61$$

$$8x+5 = 61$$

$$8x = 56$$

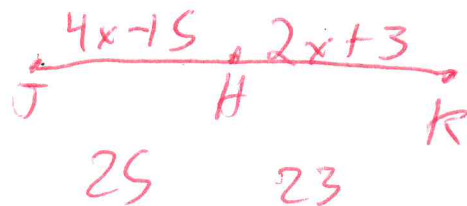
$$\boxed{x=7}$$

$$\begin{aligned} m\angle POQ &= 3x+7 \\ m\angle POQ &= 3(7)+7 \\ m\angle POQ &= 21+7 \end{aligned}$$

$$\boxed{m\angle POQ = 28}$$

$$\begin{aligned} m\angle QOR &= 5x-2 \\ m\angle QOR &= 5(7)-2 \\ m\angle QOR &= 35-2 \\ m\angle QOR &= 33 \end{aligned}$$

12. $JK = 48$ Find the value of x.



$$\begin{array}{cc} 25 & 23 \end{array}$$

$$25+23=48 \checkmark$$

$$JH + HK = JK$$

$$4x-15 + 2x+3 = 48$$

$$6x-12 = 48$$

$$\begin{array}{r} 6x = 60 \\ \hline x = 10 \end{array}$$

$$\boxed{x=10}$$

13. $M(x, y)$ is the midpoint of $\overline{CD}$ with endpoints $C(x_1, y_1)$ and $D(x_2, y_2)$

a) Find the values of x and y

b. Show $MC = MD$

$$x_m = \frac{x_1 + x_2}{2} \quad y_m = \frac{y_1 + y_2}{2}$$

$$x_m = \frac{5 + 17}{2} \quad y_m = \frac{9 + 29}{2}$$

$$x_m = \frac{22}{2} \quad y_m = \frac{38}{2}$$

$$x_m = 11 \quad y_m = 19$$

$$\boxed{(11, 19)}$$

Part 1 Find MC

$$M(11, 19) \quad C(5, 9)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(11 - 5)^2 + (19 - 9)^2}$$

$$d = \sqrt{(6)^2 + (10)^2}$$

$$d = \sqrt{36 + 100}$$

$$d = \sqrt{136}$$

$$d = \sqrt{136}$$

Part 2: Find MD

$$M(11, 19) \quad D(17, 29)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(11 - 17)^2 + (19 - 29)^2}$$

$$d = \sqrt{(-6)^2 + (-10)^2}$$

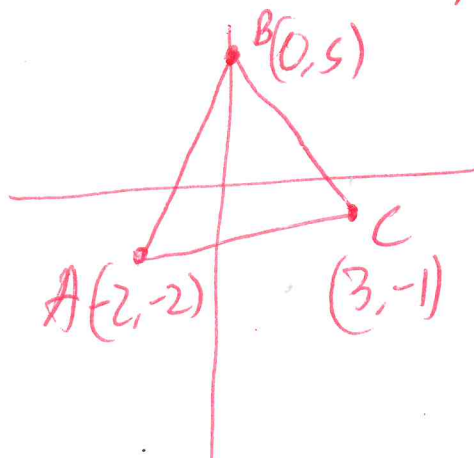
$$d = \sqrt{36 + 100}$$

$$d = \sqrt{136}$$

$$MC = MD$$

$$\sqrt{136} = \sqrt{136} \checkmark$$

14. To the nearest 10th, find the perimeter of $\triangle ABC$



AB

$$A(-2, -2) \quad B(0, 5)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2 - 0)^2 + (-2 - 5)^2}$$

$$d = \sqrt{(-2)^2 + (-7)^2}$$

$$d = \sqrt{4 + 49}$$

$$d = \sqrt{53} = 7.3$$

$$\text{Perimeter} = AB + BC + AC$$

BC $B(0, 5) \quad C(3, -1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(0 - 3)^2 + (5 - (-1))^2}$$

$$d = \sqrt{(-3)^2 + (6)^2}$$

$$d = \sqrt{9 + 36}$$

$$d = \sqrt{45} = 6.7$$

AC $A(-2, -2) \quad C(3, -1)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-2 - 3)^2 + (-2 - (-1))^2}$$

$$d = \sqrt{(-5)^2 + (-1)^2}$$

$$d = \sqrt{25 + 1}$$

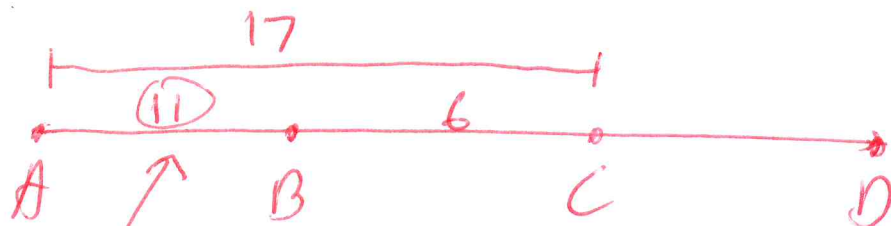
$$d = \sqrt{26} = 5.1$$

$$\text{Perimeter} = 5.1 + 7.3 + 6.7$$

$$\boxed{\text{Perimeter} = 19.1}$$

15. Let A, B, C, D be collinear and arranged in that order.

Find x if $AC = 17$ $BD = 2x - 6$ $AD = x + 16$ $BC = 6$

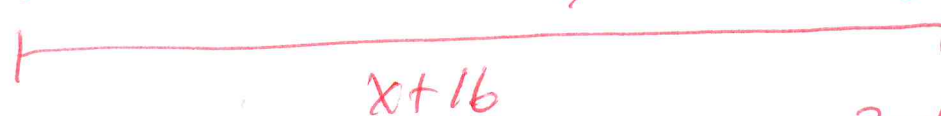
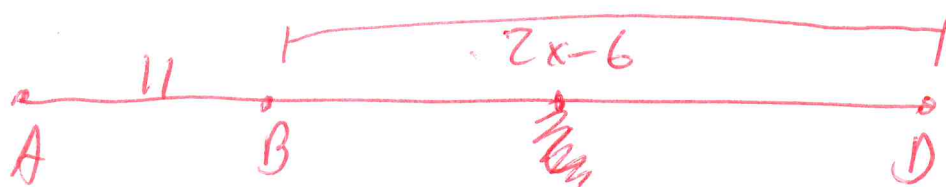


$$AB + BC = AC$$

$$AB + 6 = 17$$

$$\begin{array}{r} -6 \\ -6 \end{array}$$

$$AB = 11$$



$$2x - 6 \quad 2(11) - 6$$

$$AB + BD = AD$$

$$11 + 2x - 6 = x + 16$$

$$2x + 5 = x + 16$$

$$\begin{array}{r} -x \\ -x \end{array}$$

$$x + 5 = 16$$

$$\begin{array}{r} -5 \\ -5 \end{array}$$

$$\boxed{x = 11}$$

$$11 + 16 = 27$$

$$11 + 16 = 27$$