WHERE YOU'VE BEEN

In Chapters 1 and 2, you learned how to collect and describe data. Once the data are collected and described, you can use the results to write summaries, form conclusions, and make decisions. For instance, in the game show *Deal or No Deal,* contestants play and deal for up to \$1,000,000. By collecting and analyzing data, you can determine the chances of winning \$1,000,000.

Each game consists of 26 sealed briefcases containing dollar amounts ranging from \$.01 to \$1,000,000. Without knowing the amount in each briefcase, the contestant chooses one briefcase, which remains sealed until the end of the game. In each round, a predetermined number of the remaining briefcases are opened revealing the amount in each. At the end of each round, the "Banker" offers the contestant an amount of cash based on the amounts still left in the unopened briefcases, in exchange for the contestant's briefcase. The contestant can either accept the Banker's offer and end the game or continue to the next round. If the contestant never accepts an offer and all other briefcases have been opened, then the contestant receives what is in the briefcase he or she chose at the beginning of the game.

WHERE YOU'RE GOING

In Chapter 3, you will learn how to determine the probability that an event will occur. For instance, the probability that the first briefcase chosen by the contestant contains \$1,000,000 is $\frac{1}{26}$, because there are 26 briefcases at the start of the game and only 1 briefcase has \$1,000,000. The table below shows the probability of the contestant's briefcase containing \$1,000,000 if the briefcase with \$1,000,000 has not been opened yet.

Round		1	2	3	4	5	6	7	8	9
Total cases opened	0	6	11	15	18	20	21	22	23	24
Probability	$\frac{1}{26}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{11}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

You can also find the probability of a contestant choosing a briefcase that has at least \$100,000. (Seven briefcases have at least \$100,000.)

$$\text{Probability of at least \$100,000 in briefcase} = \frac{7}{26} \approx 0.269$$

Then, you can find the probability of a contestant choosing a briefcase that has less than \$100,000 by subtracting the probability of choosing a briefcase that has at least \$100,000 from 1.

$$\begin{aligned}\text{Probability of less than \$100,000 in briefcase} &= 1 - \text{Probability of at least \$100,000 in briefcase}\\ &= 1 - \frac{7}{26} = \frac{19}{26} \approx 0.731\end{aligned}$$

So, the probability that a contestant chooses a briefcase that contains at least \$100,000 is about 0.269, or 26.9%. The probability that a contestant chooses a briefcase that contains less than \$100,000 is about 0.731, or 73.1%.

3.1 Basic Concepts of Probability and Counting

What You SHOULD LEARN

- How to identify the sample space of a probability experiment and how to identify simple events
- How to use the Fundamental Counting Principle to find the number of ways two or more events can occur
- How to distinguish among classical probability, empirical probability, and subjective probability
- How to find the probability of the complement of an event
- How to use a tree diagram and the Fundamental Counting Principle to find more probabilities

Probability Experiments ▸ The Fundamental Counting Principle ▸ Types of Probability ▸ Complementary Events ▸ Probability Applications

▸ Probability Experiments

When weather forecasters say that there is a 90% chance of rain or a physician says there is a 35% chance for a successful surgery, they are stating the likelihood, or *probability*, that a specific event will occur. Decisions such as "should you go golfing" or "should you proceed with surgery" are often based on these probabilities. In the previous chapter, you learned about the role of the descriptive branch of statistics. Because probability is the foundation of inferential statistics, it is necessary to learn about probability before proceeding to the second branch—inferential statistics.

DEFINITION

A **probability experiment** is an action, or trial, through which specific results (counts, measurements, or responses) are obtained. The result of a single trial in a probability experiment is an **outcome.** The set of all possible outcomes of a probability experiment is the **sample space.** An **event** is a subset of the sample space. It may consist of one or more outcomes.

Study Tip

Here is a simple example of the use of the terms *probability experiment*, *sample space*, *event*, and *outcome*.

Probability Experiment:
Roll a six-sided die.

Sample Space:
{1, 2, 3, 4, 5, 6}

Event:
Roll an even number, {2, 4, 6}

Outcome:
Roll a 2, {2}

EXAMPLE 1

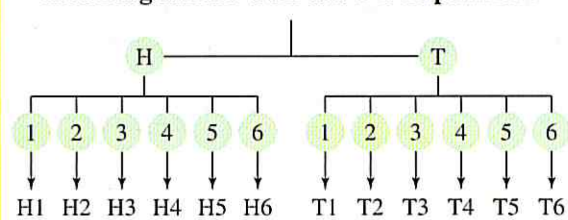
Identifying the Sample Space of a Probability Experiment

A probability experiment consists of tossing a coin and then rolling a six-sided die. Determine the number of outcomes and identify the sample space.

Solution

There are two possible outcomes when tossing a coin: a head (H) or a tail (T). For each of these, there are six possible outcomes when rolling a die: 1, 2, 3, 4, 5, or 6. One way to list outcomes for actions occurring in a sequence is to use a **tree diagram.**

Tree Diagram for Coin and Die Experiment



From the tree diagram, the sample space has 12 outcomes.

{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

SURVEY

There should be a limit to the number of terms a U.S. senator can serve.

Check one response:

☐ Agree
☐ Disagree
☐ No opinion

▸ Try It Yourself 1

For each probability experiment, determine the number of outcomes and identify the sample space.

1. A probability experiment consists of recording a response to the survey statement at the left *and* the gender of the respondent.
2. A probability experiment consists of recording a response to the survey statement at the left *and* the political party (Democrat, Republican, or Other) of the respondent.

a. Start a tree diagram by forming a branch for each possible response to the survey.
b. At the end of each survey response branch, draw a new branch for each possible outcome.
c. Find the *number of outcomes* in the sample space.
d. List the *sample space.* *Answer: Page A37*

In the rest of this chapter, you will learn how to calculate the probability or likelihood of an event. Events are often represented by uppercase letters, such as A, B, and C. An event that consists of a single outcome is called a **simple event.** In Example 1, the event "tossing heads and rolling a 3" is a simple event and can be represented as $A = \{\text{H3}\}$. In contrast, the event "tossing heads and rolling an even number" is not simple because it consists of three possible outcomes $B = \{\text{H2, H4, H6}\}$.

EXAMPLE 2

Identifying Simple Events

Determine the number of outcomes in each event. Then decide whether each event is simple or not. Explain your reasoning.

1. For quality control, you randomly select a machine part from a batch that has been manufactured that day. Event A is selecting a specific defective machine part.
2. You roll a six-sided die. Event B is rolling at least a 4.

Solution

1. Event A has only one outcome: choosing the specific defective machine part. So, the event is a simple event.
2. Event B has three outcomes: rolling a 4, a 5, or a 6. Because the event has more than one outcome, it is not simple.

▸ Try It Yourself 2

You ask for a student's age at his or her last birthday. Decide whether each event is simple or not.

1. Event C: The student's age is between 18 and 23, inclusive.
2. Event D: The student's age is 20.

a. Determine the number of outcomes in the event.
b. State whether the event is *simple* or not. *Answer: Page A37*

The Fundamental Counting Principle

In some cases, an event can occur in so many different ways that it is not practical to write out all the outcomes. When this occurs, you can rely on the Fundamental Counting Principal. The Fundamental Counting Principle can be used to find the number of ways two or more events can occur in sequence.

THE FUNDAMENTAL COUNTING PRINCIPLE

If one event can occur in m ways and a second event can occur in n ways, the number of ways the two events can occur in sequence is $m \cdot n$. This rule can be extended for any number of events occurring in sequence.

In words, the number of ways that events can occur in sequence is found by multiplying the number of ways one event can occur by the number of ways the other event(s) can occur.

EXAMPLE 3

Using the Fundamental Counting Principle

You are purchasing a new car. The possible manufacturers, car sizes, and colors are listed.

Manufacturer:	Ford, GM, Honda
Car size:	compact, midsize
Color:	white (W), red (R), black (B), green (G)

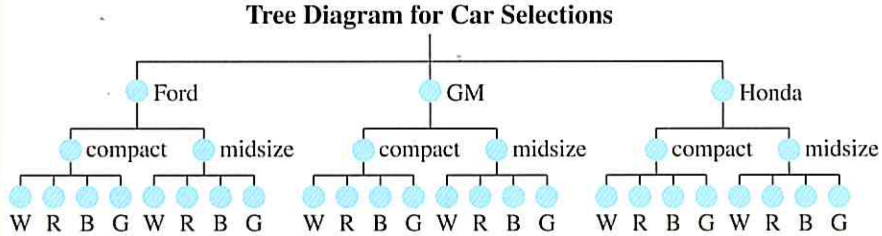
How many different ways can you select one manufacturer, one car size, and one color? Use a tree diagram to check your result.

Solution There are three choices of manufacturers, two car sizes, and four colors. Using the Fundamental Counting Principle, you can conclude that the number of ways to select one manufacturer, one car size, and one color is

$3 \cdot 2 \cdot 4 = 24$ ways.

Using a tree diagram, you can see why there are 24 options.

Tree Diagram for Car Selections



Try It Yourself 3

Your choices now include a Toyota, a large car, or a tan or gray car. How many different ways can you select one manufacturer, one car size, and one color?

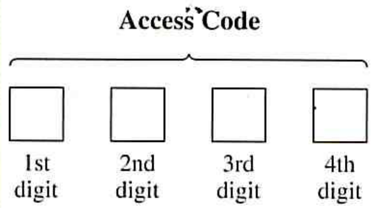
a. Find the *number of ways* each event can occur.
b. Use the *Fundamental Counting Principle.*
c. Use a *tree diagram* to check your result.

Answer: Page A37

EXAMPLE 4

Using the Fundamental Counting Principle

The access code for a car's security system consists of four digits. Each digit can be 0 through 9.



How many access codes are possible if

1. each digit can be used only once and not repeated?
2. each digit can be repeated?
3. each digit can be repeated but the first digit cannot be 0 or 1?

Solution

1. Because each digit can be used only once, there are 10 choices for the first digit, 9 choices left for the second digit, 8 choices left for the third digit, and 7 choices left for the fourth digit. Using the Fundamental Counting Principle, you can conclude that there are

$$10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

possible access codes.

2. Because each digit can be repeated, there are 10 choices for each of the four digits. So, there are

$$\begin{aligned} 10 \cdot 10 \cdot 10 \cdot 10 &= 10^4 \\ &= 10{,}000 \end{aligned}$$

possible access codes.

3. Because the first digit cannot be 0 or 1, there are 8 choices for the first digit. Then there are 10 choices for each of the other three digits. So, there are

$$8 \cdot 10 \cdot 10 \cdot 10 = 8000$$

possible access codes.

▸ Try It Yourself 4

How many license plates can you make if a license plate consists of

1. six (out of 26) alphabetical letters each of which can be repeated?
2. six (out of 26) alphabetical letters each of which cannot be repeated?
3. six (out of 26) alphabetical letters each of which can be repeated but the first letter cannot be A, B, C, or D?

a. *Identify* each event and the *number of ways* each event can occur.
b. Use the *Fundamental Counting Principle.* *Answer: Page A37*

Types of Probability

The method you will use to calculate a probability depends on the type of probability. There are three types of probability: classical probability, empirical probability, and subjective probability. The probability that event E will occur is written as $P(E)$ and is read "the probability of event E."

Study Tip

Probabilities can be written as fractions, decimals, or percents. In Example 5, the probabilities are written as fractions and decimals, rounded when necessary to three places. This *round-off rule* will be used throughout the text.

DEFINITION

Classical (or **theoretical**) **probability** is used when each outcome in a sample space is equally likely to occur. The classical probability for an event E is given by

$$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}.$$

EXAMPLE 5

Finding Classical Probabilities

You roll a six-sided die. Find the probability of each event.

1. Event A: rolling a 3
2. Event B: rolling a 7
3. Event C: rolling a number less than 5

Solution When a six-sided die is rolled, the sample space consists of six outcomes: $\{1, 2, 3, 4, 5, 6\}$.

1. There is one outcome in event $A = \{3\}$. So,

$$P(\text{rolling a 3}) = \frac{1}{6} \approx 0.167.$$

2. Because 7 is not in the sample space, there are no outcomes in event B. So,

$$P(\text{rolling a 7}) = \frac{0}{6} = 0.$$

3. There are four outcomes in event $C = \{1, 2, 3, 4\}$. So,

$$P(\text{rolling a number less than 5}) = \frac{4}{6} = \frac{2}{3} \approx 0.667.$$

Standard Deck of Playing Cards

Hearts	Diamonds	Spades	Clubs
A♥	A♦	A♠	A♣
K♥	K♦	K♠	K♣
Q♥	Q♦	Q♠	Q♣
J♥	J♦	J♠	J♣
10♥	10♦	10♠	10♣
9♥	9♦	9♠	9♣
8♥	8♦	8♠	8♣
7♥	7♦	7♠	7♣
6♥	6♦	6♠	6♣
5♥	5♦	5♠	5♣
4♥	4♦	4♠	4♣
3♥	3♦	3♠	3♣
2♥	2♦	2♠	2♣

Try It Yourself 5

You select a card from a standard deck. Find the probability of each event.

1. Event D: Selecting a seven of diamonds
2. Event E: Selecting a diamond
3. Event F: Selecting a diamond, heart, club, or spade

a. Identify the *total number of outcomes* in the sample space.
b. Find the *number of outcomes* in the event.
c. Use the *classical probability formula*.

Answer: Page A37

When an experiment is repeated many times, regular patterns are formed. These patterns make it possible to find empirical probability. Empirical probability can be used even if each outcome of an event is not equally likely to occur.

DEFINITION

Empirical (or **statistical**) **probability** is based on observations obtained from probability experiments. The empirical probability of an event E is the relative frequency of event E.

$$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}.$$

PICTURING the WORLD

It seems as if no matter how strange an event is, somebody wants to know the probability that it will occur. The following table lists the probability that some intriguing events will happen. (Adapted from Life: The Odds)

Event	Probability
Being audited by the IRS	0.6%
Writing a *New York Times* best seller	0.0045
Winning an Academy Award	0.000087
Having your identity stolen	0.5%
Spotting a UFO	0.0000003

Which of these events is most likely to occur? Least likely?

EXAMPLE 6

Finding Empirical Probabilities

A company is conducting an online survey of randomly selected individuals to determine if traffic congestion is a problem in their community. So far, 320 people have responded to the survey. The frequency distribution shows the results. What is the probability that the next person that responds to the survey says that traffic congestion is a serious problem in their community?

Response	Number of times, f
It is a serious problem.	123
It is a moderate problem.	115
It is not a problem.	82
	$\Sigma f = 320$

Solution The event is a response of "It is a serious problem." The frequency of this event is 123. Because the total of the frequencies is 320, the empirical probability of the next person saying that traffic congestion is a serious problem in their community is

$$P(\text{serious problem}) = \frac{123}{320} = 0.384.$$

▸ Try It Yourself 6

An insurance company determines that in every 100 claims, 4 are fraudulent. What is the probability that the next claim the company processes will be fraudulent?

a. *Identify* the event. Find the *frequency* of the event.
b. *Find the total frequency for the experiment.*
c. Find the *relative frequency* of the event.

Answer: Page A37

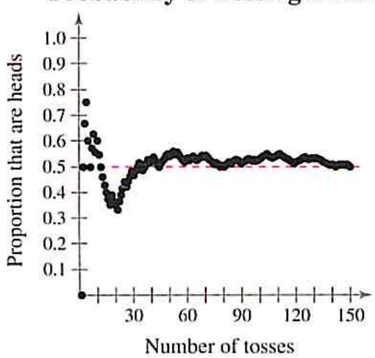
To explore this topic further, see Activity 3.1 on page 148.

As you increase the number of times a probability experiment is repeated, the empirical probability (relative frequency) of an event approaches the theoretical probability of the event. This is known as the **law of large numbers.**

LAW OF LARGE NUMBERS

As an experiment is repeated over and over, the empirical probability of an event approaches the theoretical (actual) probability of the event.

Probability of Tossing a Head



As an example of this law, suppose you want to determine the probability of tossing a head with a fair coin. If you toss the coin 10 times and get only 3 heads, you obtain an empirical probability of $\frac{3}{10}$. Because you tossed the coin only a few times, your empirical probability is not representative of the theoretical probability, which is $\frac{1}{2}$. If, however, you toss the coin several thousand times, then the law of large numbers tells you that the empirical probability will be very close to the theoretical or actual probability.

The scatter plot at the left shows the results of simulating a coin toss 150 times. Notice that, as the number of tosses increases, the probability of tossing a head gets closer and closer to the theoretical probability of 0.5.

EXAMPLE 7

Using Frequency Distributions to Find Probabilities

You survey a sample of 1000 employees at a company and record the age of each. The results are shown at the left in the frequency distribution. If you randomly select another employee, what is the probability that the employee will be between 25 and 34 years old?

Employee ages	Frequency, f
15 to 24	54
25 to 34	366
35 to 44	233
45 to 54	180
55 to 64	125
65 and over	42
	$\Sigma f = 1000$

Solution

The event is selecting an employee who is between 25 and 34 years old. In your survey, the frequency of this event is 366. Because the total of the frequencies is 1000, the probability of selecting an employee between the ages of 25 and 34 years old is

$$P(\text{age 25 to 34}) = \frac{366}{1000} = 0.366.$$

▶ Try It Yourself 7

Find the probability that an employee chosen at random will be between 15 and 24 years old.

a. Find the *frequency* of the event.
b. Find the *total of the frequencies.*
c. Find the relative frequency of the event.

Answer: Page A37

The third type of probability is **subjective probability.** Subjective probabilities result from intuition, educated guesses, and estimates. For instance, given a patient's health and extent of injuries, a doctor may feel that the patient has a 90% chance of a full recovery. Or a business analyst may predict that the chance of the employees of a certain company going on strike is 0.25.

EXAMPLE 8

Classifying Types of Probability

Classify each statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.

1. The probability that you will be married by age 30 is 0.5.
2. The probability that a voter chosen at random will vote Republican is 0.45.
3. The probability of winning a 1000-ticket raffle with one ticket is $\frac{1}{1000}$.

Solution

1. This probability is most likely based on an educated guess. It is an example of subjective probability.
2. This statement is most likely based on a survey of a sample of voters, so it is an example of empirical probability.
3. Because you know the number of outcomes and each is equally likely, this is an example of classical probability.

▸Try It Yourself 8

Based on previous counts, the probability of a salmon successfully passing through a dam on the Columbia River is 0.85. Is this statement an example of classical probability, empirical probability, or subjective probability? *(Source: Army Corps of Engineers)*

a. Identify the *event*.
b. Decide whether the probability is *determined* by knowing all possible outcomes, whether the probability is *estimated* from the results of an experiment, or whether the probability is an *educated guess*.
c. Make a *conclusion*.

Answer: Page A37

A probability cannot be negative or greater than 1. So, the probability of an event E is between 0 and 1, inclusive, as stated in the following rule.

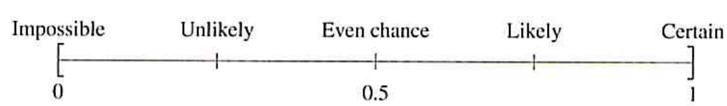
RANGE OF PROBABILITIES RULE

The probability of an event E is between 0 and 1, inclusive. That is,

$$0 \leq P(E) \leq 1.$$

If the probability of an event is 1, the event is certain to occur. If the probability of an event is 0, the event is impossible. A probability of 0.5 indicates that an event has an even chance of occurring.

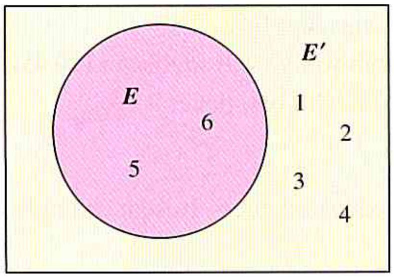
The following graph shows the possible range of probabilities and their meanings.



An event that occurs with a probability of 0.05 or less is typically considered unusual. Unusual events are highly unlikely to occur. Later in this course you will identify unusual events when studying inferential statistics.

Complementary Events

The sum of the probabilities of all outcomes in a sample space is 1 or 100%. An important result of this fact is that if you know the probability of an event E, you can find the probability of the *complement of event E*.



The area of the rectangle represents the total probability of the sample space (1 = 100%). The area of the circle represents the probability of event E, and the area outside the circle represents the probability of the complement of event E.

DEFINITION

The **complement of event E** is the set of all outcomes in a sample space that are not included in event E. The complement of event E is denoted by E' and is read as "E prime."

For instance, if you roll a die and let E be the event "the number is at least 5," then the complement of E is the event "the number is less than 5." In symbols, $E = \{5, 6\}$ and $E' = \{1, 2, 3, 4\}$.

Using the definition of the complement of an event and the fact that the sum of the probabilities of all outcomes is 1, you can determine the following formulas:

$$P(E) + P(E') = 1 \qquad P(E) = 1 - P(E') \qquad P(E') = 1 - P(E)$$

The Venn diagram illustrates the relationship between the sample space, an event E, and its complement E'.

EXAMPLE 9

Finding the Probability of the Complement of an Event

Use the frequency distribution in Example 7 to find the probability of randomly choosing an employee who is not between 25 and 34 years old.

Solution

From Example 7, you know that

$$P(\text{age 25 to 34}) = \frac{366}{1000}$$
$$= 0.366.$$

So, the probability that an employee is not between 25 and 34 years old is

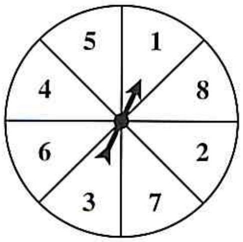
$$P(\text{age is not 25 to 34}) = 1 - \frac{366}{1000}$$
$$= \frac{634}{1000}$$
$$= 0.634.$$

Try It Yourself 9

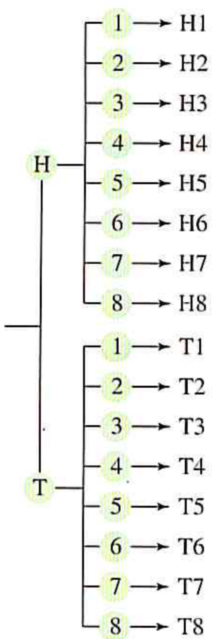
Use the frequency distribution in Example 7 to find the probability of randomly choosing an employee who is not between 45 and 54 years old.

a. *Find the probability* of randomly choosing an employee who is between 45 and 54 years old.
b. *Subtract* the resulting probability from 1.
c. State the probability as a fraction and as a decimal.

Answer: Page A37



Tree Diagram for Coin and Spinner Experiment



▸ Probability Applications

EXAMPLE 10

Using a Tree Diagram

A probability experiment consists of tossing a coin and spinning the spinner shown to the left. The spinner is equally likely to land on each number. Use a tree diagram to find the probability of each event.

1. Event A: tossing a tail and spinning an odd number
2. Event B: tossing a head or spinning a number greater than 3

Solution From the tree diagram to the left, you can see that there are 16 outcomes.

1. There are four outcomes in event $A = \{T1, T3, T5, T7\}$. So,

$$P(\text{tossing a tail and spinning an odd number}) = \frac{4}{16} = \frac{1}{4} = 0.25.$$

2. There are 13 outcomes in event $B = \{H1, H2, H3, H4, H5, H6, H7, H8, T4, T5, T6, T7, T8\}$. So,

$$P(\text{tossing a head or spinning a number greater than 3}) = \frac{13}{16} \approx 0.813.$$

▸ Try It Yourself 10

Find the probability of tossing a tail and spinning a number less than 6.

a. Determine the *total number* of outcomes.
b. Find the number of outcomes in the *event*.
c. Find the *probability of the event*.

Answer: Page A37

EXAMPLE 11

Using the Fundamental Counting Principle

Your college identification number consists of 8 digits. Each digit can be 0 through 9 and each digit can be repeated. What is the probability of getting your college identification number when randomly generating eight digits?

Solution Because each digit can be repeated, there are 10 choices for each of the 8 digits. So, using the Fundamental Counting Principle, there are $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^8 = 100{,}000{,}000$ possible identification numbers.

But only one of those numbers corresponds to your college identification number. So, the probability of randomly generating 8 digits and getting your college identification number is $1/100{,}000{,}000$.

▸ Try It Yourself 11

Your college identification number consists of 9 digits. The first two digits of each number will be the last two digits of the year you graduate. The other digits can be 0 through 9 and each digit can be repeated. What is the probability of getting your college identification number when randomly generating the other seven digits?

a. Find the *total number* of possible identification numbers. Assume that you are scheduled to graduate in 2012.
b. Find the *probability* of randomly generating your identification number.

Answer: Page A37

3.1 EXERCISES

Building Basic Skills and Vocabulary

1. Determine which of the following numbers could not represent the probability of an event. Explain your reasoning.

(a) 0 (b) 0.001 (c) -1 (d) 50% (e) $\frac{745}{1262}$ (f) $\frac{45}{31}$

2. Explain why the following statement is incorrect:

The probability of rain tomorrow is 150%.

3. When you use the Fundamental Counting Principle, what are you counting?

4. Use your own words to describe the law of large numbers. Give an example.

Identifying a Sample Space *In Exercises 5–8, identify the sample space of the probability experiment and determine the number of outcomes in the sample space. Draw a tree diagram if it is appropriate.*

5. Guessing the initial of a student's middle name

6. Tossing three coins

7. Determining a person's blood type (A, B, AB, O) and Rh-factor (positive, negative)

8. Rolling a pair of six-sided dice

Recognizing Simple Events *In Exercises 9–12, determine the number of outcomes in each event. Then decide whether the event is a simple event or not. Explain your reasoning.*

9. A computer is used to randomly select a number between 1 and 2000. Event A is selecting 359.

10. A computer is used to randomly select a number between 1 and 2000. Event B is selecting a number less than 200.

11. You randomly select one card from a standard deck. Event A is selecting a king.

12. You randomly select one card from a standard deck. Event B is selecting a four of hearts.

13. Job Openings An insurance company is hiring for two positions: an actuary and a claims adjuster. How many ways can these positions be filled if there are 9 people applying for the actuarial position and 15 people applying for the claims adjuster position?

14. Menu A menu has three choices for salad, six main dishes, and four desserts. How many different meals are available if you select a salad, a main dish, and a dessert?

15. Security System The access code for a car's security system consists of four digits. The first digit cannot be zero and the last digit must be odd. How many different codes are available?

16. True or False Quiz Assuming that no questions are left unanswered, in how many ways can a six-question true-false quiz be answered?

True or False? *In Exercises 17–20, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

17. If you roll a six-sided die six times, you will roll an even number at least once.

18. You flip a fair coin nine times and it lands tails up each time. The probability it will land heads up on the tenth flip is greater than 0.5.

19. A probability of 0.25 indicates an unusual event.

20. If an event is almost certain to happen, its complement will be an unusual event.

Matching Probabilities *In Exercises 21–24, match the event with its probability.*

(a) 0.95 (b) 0.05 (c) 0.25 (d) 0

21. You toss a coin and randomly select a number from 0 to 9. What is the probability of getting tails and selecting a 3?

22. A random number generator is used to select a number from 1 to 100. What is the probability of selecting the number 153?

23. A game show contestant must randomly select a door. One door doubles her money while the other three doors leave her bankrupt. What is the probability she selects the door that doubles her money?

24. Five of the 100 DVD players in an inventory are known to be defective. What is the probability you randomly select an item that is not defective?

Classifying Types of Probability *In Exercises 25 and 26, classify the statement as an example of classical probability, empirical probability, or subjective probability. Explain your reasoning.*

25. According to company records, the probability that a washing machine will need repairs during a six-year period is 0.10.

26. The probability of choosing 6 numbers from 1 to 40 that match the 6 numbers drawn by a state lottery is $1/3,838,380 \approx 0.00000026$.

Finding Probabilities *In Exercises 27–30, consider a company that selects employees for random drug tests. The company uses a computer to randomly select employee numbers that range from 1 to 6296.*

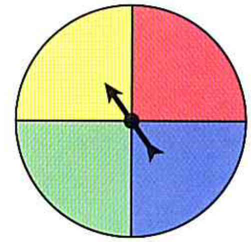
27. Find the probability of selecting a number less than 1000.

28. Find the probability of selecting a number greater than 1000.

29. Find the probability of selecting a number divisible by 1000.

30. Find the probability of selecting a number that is not divisible by 1000.

Probability Experiment *In Exercises 31–34, a probability experiment consists of rolling a six-sided die and spinning the spinner shown. The spinner is equally likely to land on each color. Use a tree diagram to find the probability of each event.*



31. Event A: rolling a 5 and the spinner landing on blue

32. Event B: rolling an odd number and the spinner landing on green

33. Event C: rolling a number less than 6 and the spinner landing on yellow

34. Event D: not rolling a number less than 6 and the spinner landing on yellow

35. Security System The access code for a garage door consists of three digits. Each digit can be 0 through 9 and each digit can be repeated.

(a) Find the number of possible access codes.

(b) What is the probability of randomly selecting the correct access code?

(c) What is the probability of not selecting the correct access code?

36. Security System An access code consists of a letter followed by four digits. Any letter can be used, the first digit cannot be 0, and the last digit must be even.

(a) Find the number of possible access codes.

(b) What is the probability of randomly selecting the correct access code on the first try?

(c) What is the probability of not selecting the correct access code on the first try?

Using and Interpreting Concepts

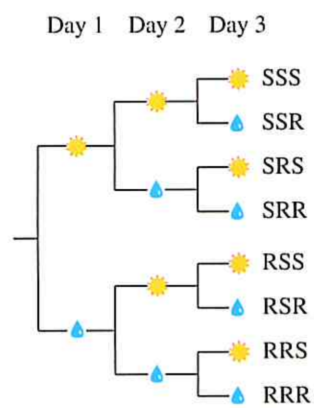
Wet or Dry? *You are planning a three-day trip to Seattle, Washington, in October. Use the following tree diagram to answer each question.*

37. List the sample space.

38. List the outcome(s) of the event "It rains all three days."

39. List the outcome(s) of the event "It rains on exactly one day."

40. List the outcome(s) of the event "It rains on at least one day."



41. Sunny and Rainy Days You are planning a four-day trip to Seattle, Washington, in October.

(a) Make a sunny day/rainy day tree diagram for your trip.

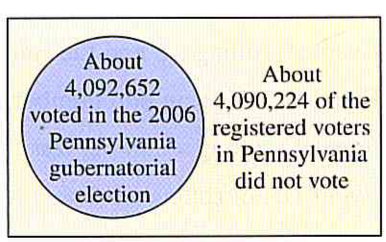
(b) List the sample space.

(c) List the outcome(s) of the event "It rains on exactly one day."

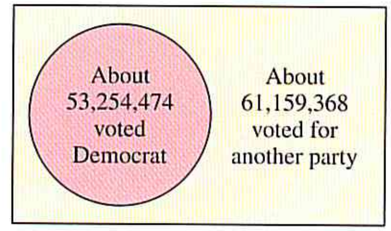
42. Machine Part Suppliers Your company buys machine parts from three different suppliers. Make a tree diagram that shows the three suppliers and whether the parts they supply are defective.

Graphical Analysis *In Exercises 43 and 44, use the diagram to answer the question.*

43. What is the probability that a registered voter in Pennsylvania voted in the 2006 gubernatorial election? *(Source: Pennsylvania Department of State)*



44. What is the probability that a voter chosen at random did not vote for a Democratic representative in the 2004 election? *(Source: Federal Election Commission)*



Using a Frequency Distribution to Find Probabilities *In Exercises 45–48, use the frequency distribution, which shows the number of American voters (in millions) according to age. (Source: U.S. Bureau of the Census)*

Ages of voters	Frequency (in millions)
18 to 20 years old	5.8
21 to 24 years old	8.5
25 to 34 years old	21.7
35 to 44 years old	27.7
45 to 64 years old	51.7
65 years old and over	26.7

Find the probability that a voter chosen at random is

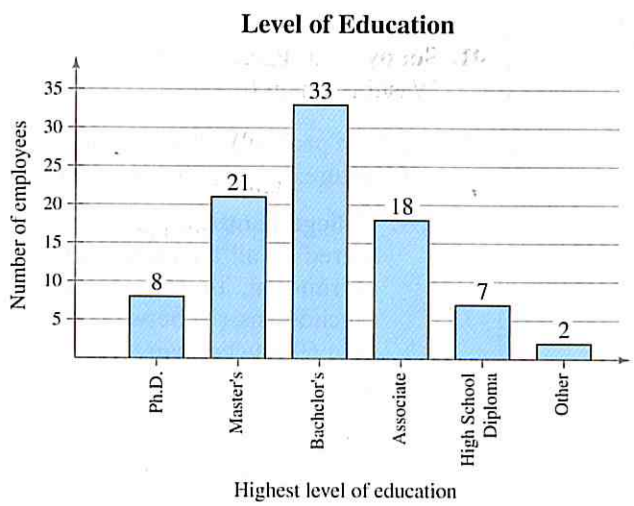
45. between 21 and 24 years old.

46. between 35 and 44 years old.

47. not between 18 and 20 years old.

48. not between 25 and 34 years old.

Using a Bar Graph to Find Probabilities *In Exercises 49–52, use the following bar graph, which shows the highest level of education received by employees of a company.*



Find the probability that the highest level of education for an employee chosen at random is

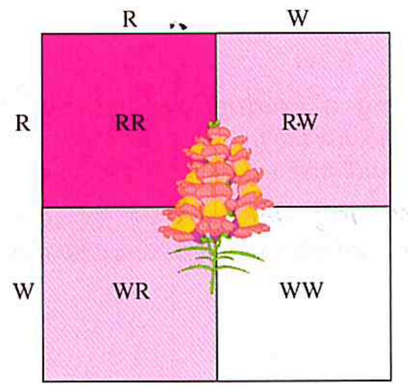
49. a Ph.D.

50. an Associate degree.

51. a Master's degree.

52. a Bachelor's degree.

53. Genetics When two pink snapdragon flowers (RW) are crossed, there are four equally likely possible outcomes for the genetic makeup of the offspring: red (RR), pink (RW), pink (WR), and white (WW). If two pink snapdragons are crossed, what is the probability that the offspring will be (a) pink, (b) red, and (c) white?



Parents
Ssmm and SsMm

	SM	Sm
Sm	SSMm	SSmm
Sm	SSMm	SSmm
sm	SsMm	Ssmm
sm	SsMm	Ssmm

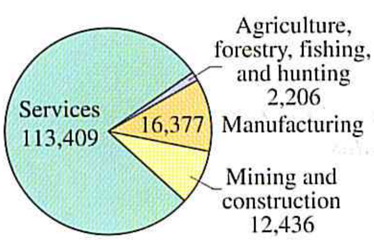
	sM	sm
Sm	SsMm	Ssmm
Sm	SsMm	Ssmm
sm	ssMm	ssmm
sm	ssMm	ssmm

54. Genetics There are six basic types of coloring in registered collies: sable (SSmm), tricolor (ssmm), trifactored sable (Ssmm), blue merle (ssMm), sable merle (SSMm), and trifactored sable merle (SsMm). The *Punnett square* at the left shows the possible coloring of the offspring of a trifactored sable merle collie and a trifactored sable collie. What is the probability that the offspring will have the same coloring as one of its parents?

Using a Pie Chart to Find Probabilities

In Exercises 55–58, use the pie chart at the left, which shows the number of workers (in thousands) by industry for the United States. *(Source: U.S. Bureau of Labor Statistics)*

Workers (in thousands) by Industry for the U.S.



55. Find the probability that a worker chosen at random was employed in the services industry.

56. Find the probability that a worker chosen at random was employed in the manufacturing industry.

57. Find the probability that a worker chosen at random was not employed in the services industry.

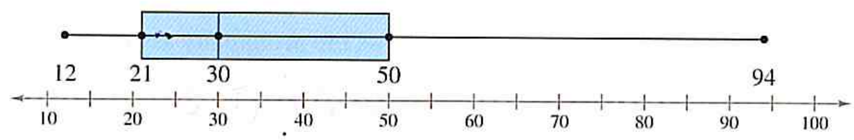
58. Find the probability that a worker chosen at random was not employed in the agriculture, forestry, fishing, and hunting industry.

59. College Football A stem-and-leaf plot for the number of touchdowns scored by all Division 1A football teams is shown. If a team is selected at random, find the probability the team scored (a) at least 31 touchdowns, (b) between 40 and 50 touchdowns inclusive, and (c) more than 69 touchdowns.

1	5 5 5 7 8 8
2	0 1 1 1 1 2 2 3 4 4 5 5 6 6 7 7 7 8 8 8 9 9 9
3	0 0 1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 4 4 4 4 5 5 5 5 5 6 6 7 7 7 8 8 8 9 9 9
4	0 0 0 1 1 2 2 3 3 3 4 4 4 4 5 5 5 5 6 6 6 6 7 7 8 8 8 8 8 9
5	0 0 2 3 3 4 4 5 5 5 5 7 9 9
6	0 1 1 1 3 3 5 8
7	
8	9

Key: 1|5 = 15

60. Individual Stock Price An individual stock is selected at random from the portfolio represented by the box-and-whisker plot shown. Find the probability that the stock price is (a) less than \$21, (b) between \$21 and \$50, and (c) \$30 or more.



Writing *In Exercises 61 and 62, write a statement that represents the complement of the given probability.*

61. The probability of randomly choosing a tea drinker who has a college degree (Assume that you are choosing from the population of all tea drinkers.)

62. The probability of randomly choosing a smoker whose mother also smoked (Assume that you are choosing from the population of all smokers.)

Extending Concepts

63. Rolling a Pair of Dice You roll a pair of six-sided dice and record the sum.

(a) List all of the possible sums and determine the probability of rolling each sum.

(b) Use a technology tool to simulate rolling a pair of dice and recording the sum 100 times. Make a tally of the 100 sums and use these results to list the probability of rolling each sum.

(c) Compare the probabilities in part (a) with the probabilities in part (b). Explain any similarities or differences.

Odds *In Exercises 64–67, use the following information. In gambling, the chances of winning are often written in terms of odds rather than probabilities. The* ***odds of winning*** *is the ratio of the number of successful outcomes to the number of unsuccessful outcomes. The* ***odds of losing*** *is the ratio of the number of unsuccessful outcomes to the number of successful outcomes. For example, if the number of successful outcomes is 2 and the number of unsuccessful outcomes is 3, the odds of winning are 2 : 3 (read "2 to 3") or $\frac{2}{3}$. (Note: The probability of success is $\frac{2}{5}$.)*

64. A beverage company puts game pieces under the caps of its drinks and claims that one in six game pieces wins a prize. The official rules of the contest state that the odds of winning a prize are 1 : 6. Is the claim "one in six game pieces wins a prize" correct? Why or why not?

65. The odds of an event occurring are 4 : 5. Find (a) the probability that the event will occur and (b) the probability that the event will not occur.

66. A card is picked at random from a standard deck of 52 playing cards. Find the odds that it is a spade.

67. A card is picked at random from a standard deck of 52 playing cards. Find the odds that it is not a spade.