Proving Polynomial Identities

Notes

Polynomial identity: when one polynomial equals another

ex: $(x+a)(x+b) = x^2 + ax + bx + ab$

Can prove 2 ways: numerical verification
algebraic proof

Numerical Verification

1. Pick any number for each variable and substitute
2. Simplify each side
3. Show that the sides are equal

Don't use 0, ±1

Example 1

Verify $(x+a)(x+b) = x^2 + ax + bx + ab$

let $x=2, a=3, b=4$

$(2+3)(2+4) \stackrel{?}{=} 2^2 + 3(2) + 4(2) + 3(4)$

$(5)(6) \stackrel{?}{=} 4 + 6 + 8 + 12$

$30 \stackrel{?}{=} 30$

$30 = 30$ ✓

Algebraic Proof

1. Pick one side to work on. → FOIL, factor, simplify, combine like terms, etc
2. Make that side look like the other side.

Example 2

Prove $(x+a)(x+b) = x^2 + ax + bx + ab$

↑ side I'm working on

$(x+a)(x+b)$

$x^2 + bx + ax + ab$

$x^2 + ax + bx + ab = x^2 + ax + bx + ab$ ✓

(I foiled, then switched 2 terms around)

right side

(left side now matches right side)