

T/PC
Final Exam
Study Guide 5.1-5.3 circled only

$$1. \frac{\csc x - 1}{\csc x + 1} = \frac{\cot^2 x}{\csc^2 x + 2\csc x + 1}$$

$$\text{Goal} \quad \frac{\cot^2 x}{(\csc x + 1)^2} = \frac{\csc^2 x - 1}{(\csc^2 x + 1)^2} = \frac{(\csc x + 1)(\csc x - 1)}{(\csc x + 1)(\csc x + 1)} = \frac{\csc x - 1}{\csc x + 1}$$

$$3. \tan^5 x = \tan x (\sec^4 x - 2\sec^2 x + 1) \leftarrow \text{Goal}$$

$$\rightarrow \tan^5 x = \tan x (\tan^4 x) = \tan x (\tan^2 x)(\tan^2 x) = \tan x (\sec^2 x - 1)^2$$

$$= \tan x (\sec^4 x - 2\sec^2 x + 1)$$

$$4. \frac{\cot x}{\csc x + 1} = \frac{\csc x - 1}{\cot x} \leftarrow \text{goal}$$

$$\downarrow$$

$$\frac{\cot x}{\csc x + 1} \cdot \frac{\csc x - 1}{\csc x - 1} = \frac{\cot x (\csc x - 1)}{\csc^2 x - 1} = \frac{\cot x (\csc x - 1)}{\cot^2 x} = \frac{\csc x - 1}{\cot x}$$

$$6. \cot^2 x + \csc^2 x = 2\csc^2 x - 1 \leftarrow \text{goal}$$

$$\downarrow$$

$$\cot^2 x + \csc^2 x = \csc^2 x - 1 + \csc^2 x = 2\csc^2 x - 1$$

$$9. \frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = \frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)} + \frac{1 - \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{2}{1 - \cos^2 x} = \frac{2}{\sin^2 x} = 2\csc^2 x$$

$$11. \frac{(\csc y + \cot y)(\csc y - \cot y)}{\csc y} = \frac{\csc^2 y - \cot^2 y}{\csc y} = \frac{1}{\csc y} = \sin y$$

(12. same as 1 from 1st pg)

$$13. \cos \theta \text{ if } \sin \theta = -\frac{12}{13} \text{ and } \tan \theta > 0 \text{ QIII}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\text{QIII: } \cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$\cos \theta = -\sqrt{1 - \left(-\frac{12}{13}\right)^2}$$

$$\cos \theta = -\sqrt{1 - \frac{144}{169}}$$

$$\cos \theta = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$