Chapter 5 Day 1: Trig Identities

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\cot\theta = \frac{1}{\tan\theta}$
3. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
4. $\tan\theta = \frac{1}{\cot\theta}$
5. $\sec\theta = \frac{1}{\cos\theta}$
6. $\cos\theta = \frac{1}{\sec\theta}$
7. $\csc\theta = \frac{1}{\sin\theta}$
8. $\sin\theta = \frac{1}{\csc\theta}$

9. $\cos^2\theta + \sin^2\theta = 1$ *
10. $\sin^2\theta = 1 - \cos^2\theta$
11. $\cos^2\theta = 1 - \sin^2\theta$
12. $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ (divide by $\cos^2\theta$)

$$1 + \tan^2\theta = \sec^2\theta$$

13. $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ (divide by $\sin^2\theta$)

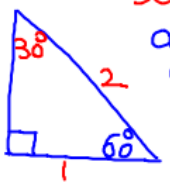
$$\cot^2\theta + 1 = \csc^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$\sin(90-60) = \cos 60^\circ$

14. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ ($\frac{\pi}{2} - \theta$ and θ are complimentary)
15. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
16. $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
17. $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$
18. $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$
19. $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

30° & 60° are complimentary



$\cos 60^\circ = \frac{1}{2}$

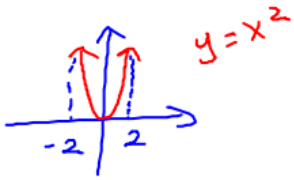
$\sin 30^\circ = \frac{1}{2}$

the cos and sin of complimentary angles are equal

Odd/Even Functions.

Even

20. $\cos(-\theta) = \cos(\theta)$
21. $\sec(-\theta) = \sec(\theta)$



$2 \xrightarrow{2^2} 4$

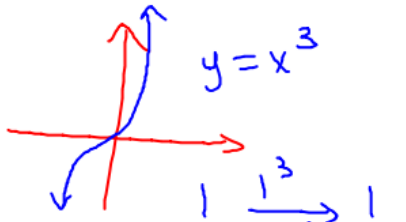
$-2 \xrightarrow{(-2)^2} 4$

opposite input → same output

Odd.

22. $\sin(-\theta) = -\sin(\theta)$
23. $\csc(-\theta) = -\csc(\theta)$
24. $\tan(-\theta) = -\tan\theta$
25. $\cot(-\theta) = -\cot\theta$

opposite input → opposite output.



$1 \xrightarrow{1^3} 1$

$-1 \xrightarrow{(-1)^3} -1$

$\frac{12}{4} = 3$

Sample Problems

Simplify the following trig expressions.

Ex 1: $\sin^3 x + \sin x \cos^2 x$

$= \sin x \sin^2 x + \sin x \cos^2 x$

$= \sin x (\underbrace{\sin^2 x + \cos^2 x}_{1})$ (#9)

$= \sin x$

Ex 2: $\frac{(a+b)(a-b)}{\sin^2 x} = \frac{a^2 - b^2}{\sin^2 x}$ #12

$= \frac{\sec^2 x - 1}{\sin^2 x}$

$= \frac{\tan^2 x}{\sin^2 x}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{1}}$ (#1)

KCF $= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x$ (#5)

Ex 3: $\frac{\cos x \cos x}{\cos x(1-\sin x)} - \frac{\sin x(1-\sin x)}{(\cos x)(1-\sin x)}$

$= \frac{\cos^2 x}{\cos(1-\sin x)} - \frac{\sin x - \sin^2 x}{\cos x(1-\sin x)}$

$= \frac{\cos^2 x - \sin x + \sin^2 x}{\cos x(1-\sin x)}$ #9

$= \frac{1-\sin x}{\cos x(1-\sin x)}$

$= \frac{1}{\cos x} = \sec x$

#5

Ex 4: $\cos(-x) - \cos(x)$ #20

$= 0$

Hw p 410 # 9-22