Final study guide S, key.

Final Exam Objectives- with examples	Hints
1. Exponent Rules: Review table p7 Properties of exponents (p10 #47-52). Example: $\frac{x^4y^3}{x^2y^5}$ $x^2y^{-2} = \frac{x^2}{y^2}$ $\left(\frac{x}{y}\right)^2$ Example: $\left(\frac{4a^3b}{a^2b^3}\right)\left(\frac{3b^2}{2a^2b^4}\right)$ $\frac{4a}{b^2} \cdot \frac{3}{2a^2b} = \frac{12a}{2a^2b^4} = \frac{6}{ab^4}$	
2. Find the standard form equation of a circle definition p15 (p19#41-48) Example: Center (1,2), radius 5 $(x-1)^2+(y-2)^2 = 5^2$ Example: Center (0,0), radius $\sqrt{3}$ $(x-0)^2+(y-0)^2 = \sqrt{3}^2$ $x^2+y^2=3$	$(x-h)^2+(y-k)^2=r^2$ center (h,k)
3. Find a point slope form equation for the line through the point with the given slope. Forms of equations of lines p30 (p36#11- 14). Example: Point (1,4) Slope m=2 $y-4=2(x-1)$ Example: Point (-3,4) Slope m=3 $y-4=3(x+3)$	$y-y_1=m(x-x_1)$ if choices are $y=mx+b$, distribute
4. Solve quadratic inequality algebraically, notes p55. (p 58 #9-14). Example: $2x^2+17x+21 \leq 0$ ← less than or equal to $(x+7)(2x+3)$ + − + ; −7, −1.5 Example: $21+4x-x^2>0$	Sol: $[-7, -\frac{3}{2}]$
5. Find the domain of a function from the equation, notes p82 (p95#9-14)	

divide by −

$x^2+4x+21 > 0$ positive

$x^2-4x-21<0$

$(x-3)(x+7)>0$

$(-\infty,-3)\cup(7,\infty)$

−21; +3, −7, −4

$(x+3)(x-7)>0$

+ − + ; −3, 7

p1 - key

+ − − + ; $-\infty$, −4, 0, 4, ∞

$(-\infty,-4]\cup\{0\}\cup[4,\infty)$

Example: $f(x) = x^2+4$ $(-\infty,\infty)$ Example: $f(x)=\sqrt{x^4-16x^2}$ $x^2(x^2-16)$ always + $x^2(x-4)(x+4)$	what's under the square root has to be +
6. Identify increasing and decreasing intervals of functions (p95#29-34). Example: $f(x) = \|x+2\|-1$ ↗ $[-2,\infty)$ ↘ $(-\infty,-2]$ Example: $f(x) = x^3-x^2-2x$ ↗ $\begin{cases}(-\infty,-0.549)\\(1.215,\infty)\end{cases}$ ↘	Graph them $(-0.549, 1.215)$
7. Find the vertical and Horizontal asymptotes of a function notes p93 (p95#55-62). Example: $f(x) = \frac{x}{x-1}$ H.A $y=1$ $x-1=0$ $x=1$ V.A Example: $h(x) = \frac{2x-4}{x^2-4}$ H.A $y=0$ x^2-4 $x=2$ hole $(x-2)(x+2)$ $x=-2$ V.A	hole value that makes both N & D equal to 0 V.A value that makes D = 0
8. Composition of functions p 111 (p117#15-19). Example: $f(x) = 3x+2;\ g(x)=x-1$ $f\circ g = f(g(x)) = 3(x-1)+2$ $= 3x-3+2$ $= 3x-1$ Example: $f(x)=x^2;\ g(x)=\sqrt{1-x^2}$ $f\circ g(x) = f(g(x)) = (\sqrt{1-x^2})^2 = 1-x^2$	$f\circ g$ put g in f = $f(g(x))$

$g\circ f(x) = g(f(x)) = \sqrt{1-(x^2)^2} = \sqrt{1-x^4}$

$g\circ f(x) = g(f(x)) = 3x+2-1$

$= 3x+1$

put f in g

$x = \frac{y+3}{y-2} \rightarrow xy - 2x = y + 3$

$xy - y = 2x + 3$

$y(x-1) = 2x + 3 \rightarrow y = \frac{2x+3}{x-1}$

9. Find the inverse of a function p123 (p126#13-16).

Example: $f(x) = 3x - 6$

switch variables: $y = 3x - 6$, $x = 3y - 6$

solve for y: $\frac{3y}{3} = \frac{x+6}{3}$

$y = \frac{1}{3}x + 2$

Example: $f(x) = \frac{x+3}{x-2}$

$y = \frac{x+3}{x-2}$

10. Transformations p136#9-12

Example: $y = -\sqrt{x}$

Reflection across x-axis

Example: $y = \sqrt{3 - x}$ ← 3 to the right, reflection across y-axis (green graph above)

11. Modeling with functions section 1.7 (p149#23,31,34).

Example: Mark received a 3.5% salary increase. His salary after the raise was $36,432. What was his salary before the raise?

$x + 3.5\%(x) = 36,432$ ← final salary

$1.035x = 36,432$ $x = 35,200$

Before x, after 36,432, 3.5%

5 increase, 36,432 after, ? original

Example: How much 10% solution and how much 45% solution should be mixed together to make 100 gallons of 25% solution?

$0.1x + 0.45(100 - x) = 0.25(100)$

solve for X $0.1x + 45 - 0.45x = 25$

$x = 57.14$

10% x, 45% $100 - x$, 100 gal, 25% solution

$100 - 57.14 = 42.86$ gallons

we don't add %.

12. Find the vertex of a quadratic function from vertex form and standard form; then rewrite in vertex form (p165-166 example 5-6, p169#23-32)

Example: $f(x) = -3x^2 + 6x - 5$ (a, b, c)

$h = \frac{-6}{-6} = 1$

$f(1) = -3(1)^2 + 6(1) - 5$

$k = -2$

$f(x) = -3(x-1)^2 - 2$

plug h in f(x) to find k

Example: $g(x) = 2(x - \sqrt{3})^2 + 4$

already in vertex form

$h(\sqrt{3}, 4)$

13.Modeling with Quadratics (p170#56, p171#61-62)

Example: A large painting in the style of Rubens is 3 ft longer than it is wide. If the wooden frame is 12 in. wide, the area of the picture and frame is 208 ft^2, find the dimensions of the painting.

$w = 11$ ft Length = 14 ft

12, w+3, 12, w, 12 in, 12, 12 in

Area = L x W

$208 = (w+3+2)(w+2)$ $w^2 + 7w - 198 = 0$

Length = 11+3 = 14 ft

solve it by graphing or algebraicly (x-meth

Example: As a promotion for the Houston Astros downtown ballpark, a competition is held to see who can throw a baseball the highest from the front row of the upper deck of seats, 83 ft above field level. The winner throws the ball with an initial vertical velocity of 92 ft/sec and it lands on the infield grass.

(a) Find the maximum height of the baseball.

(b) How much time is the ball in the air?

(c) Determine its vertical velocity when it hits the ground.

14. Finding zeros of polynomial functions by graphing and algebraically (p193#33-38, p194#43-48)

Example: $f(x) = x^2 + 2x - 8$

Finding zeros → graphing, → algebrically

$(x-2)(x+4)$

$x = 2, -4$

Menu - analyze graph - zeros

Example: $f(x) = 2x^5 - 11x^4 + 4x^3 + 47x^2 - 42x - 8$

Graph →

$x = -1.97, -.162, 1.25, 2.77, 3.62$

15. Synthetic division (p205#7-10), Remainder theorem (p205#13-18) and Factor theorem (p205#19-24).

Example: Synthetic ÷; $f(x) = \frac{x^3 - 5x^2 + 3x - 2}{x+1}$

$-1 \,|\, 1 \;\; -5 \;\; 3 \;\; -2$

$\quad\;\; -1 \;\; 6 \;\; -9$

$\quad\; 1 \;\; -6 \;\; 9 \;\; -11$

Fraction Form $= x^2 - 6x + 9 - \frac{11}{x+1}$

Polynomial Form = (quotient)(divisor) + R

$= (x^2 - 6x + 9)(x+1) - 11$

$= (x-3)^2(x+1) - 11$

Example: Remainder Theorem; $f(x) = x^4 - 5$; $k = 1$

$(1)^4 - 5 = -4$

Example: $x - 1$; $x^3 - x^2 + x - 1$

$(1)^3 - (1)^2 + (1) - 1 = 0$

you just plug in K, from x-K

If Remaindor is zero then its a factor

- p2 - Key

16. Solving Rational Equations (p232#1-18).

Example: $\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}$

$x-2+x+5=1$

$2x+3=3$

$\frac{2x}{2}=\frac{0}{2}$

$\boxed{x=0}$

Example: $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

$\frac{(x+3)^2}{x(x+3)} - \frac{2x}{x(x+3)} = \frac{6}{x^2+3x}$

$x^2+6x+9-2x=6$

$x^2+4x+9=6$

$x^2+4x+3=0$

$(x+1)(x+3)=0$

$x=-1$

Example: $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2+3x-4}$

$\frac{4x(x-1)}{(x+4)(x-1)} + \frac{3(x+4)}{(x-1)(x+4)} = \frac{15}{x^2+3x-4}$

$4x^2-4x+3x+12-15=0$

$x^2-x-3=0$

$=-0.75$

17. Solving Rational Inequality (p243#33-38).

Example: $\frac{x-1}{x^2-4} < 0$ → negative

$\frac{x-1}{(x-2)(x+2)}$

− und + − und +

$-2 \quad 1 \quad 2$

S: $(-\infty,-2) \cup (1,2)$

Example: $\frac{x^2+3x-10}{x^2-6x+9} < 0$ → negative

$\frac{(x+5)(x-2)}{(x-3)(x-3)}$

+ − + und +

$-5 \quad 2 \quad 3$

S: $(-5,2)$

Rational inqualities
- zeros
- und
- numberline
- choose numbers
- look at true ones (signs)

18. Modeling with exponential functions (p271#7-18, 29-34)

Example: Initial Value= 5, Increasing at a rate of 17% per year

$y=5(1+0.17)^x$

$y=5(1.17)^x$

$a=5$

$r=17\%=0.17$

$y=ab^x$

Example: The 2000 population of Jacksonville, Florida, was 736,000 and was increasing at the rate of 1.49% each year. At that rate, when will the population be 1 million?

$1000000=736{,}000(1.0149)^x$

$x=20.725 \approx x=21$ yrs.

$2000 \to \underbrace{736{,}000}_{a}$ 1million ← final amount

$A=ab^x$

$r=1.49\%$

$=0.0149\%$

19. Properties of log p289#1-22

Example: ln8x

$=\ln 8+\ln x$

$=\ln 2^3+\ln x = 3\ln 2+\ln x$

Example: $\log \sqrt[4]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/4}$

$=\frac{1}{4}\log\frac{x}{y} = \frac{1}{4}\log x - \frac{1}{4}\log y$

Example: $4log(xy) - 3log(yz)$

$\log(xy)^4 - \log(yz)^3$

$\log \frac{x^4y^4}{y^3z^3} = \log\frac{x^4y}{z^3}$

20. Solving equations with log (p301#25-28)

Example: $logx^2 = 6$

$10^6 = x^2$

$\sqrt{10^6}=\sqrt{x^2}$

$x=1000$

$x=-1000$

Example: $lnx^6 = 12$

$e^{12}=x^6$

$\sqrt[6]{e^{12}}=\sqrt[6]{x^6}$

$x=\pm e^2$

Example: $logx - \frac{1}{2}log(x+4) = 1$

$\log x - \log\sqrt{x+4} = \log 10$

$\log\frac{x}{\sqrt{x+4}} = \log 10$

$(x)^2=(10\sqrt{x+4})^2$

$x^2=100(x+4)$

$x^2=100x+400$

$x^2-100x-400=0$

Graph it and find zeros.

$x=103.85$

Example: $ln(x+3) + ln(x+4) = 3ln2$

$\ln(x+3)(x+4)=\ln 2^3$

$\ln(x^2+4x+3x+12)=\ln 8$

$\cancel{\ln}(x^2+7x+12)=\cancel{\ln}8$

$x^2+7x+12=8$

$x^2+7x+4=0$

$x=\cancel{-6.37}$ ← makes input negative ln(x+3)

21. Interest problems, formulas p306 (p301#1-8, p311 #21-22,25,26)

Example: $36(\frac{1}{3})^{x/5} = 4$

$\left(\frac{1}{3}\right)^{x/5}=\frac{1}{9}$

$\left(\frac{1}{3}\right)^{x/5}=\left(\frac{1}{3}\right)^2$

$\frac{x}{5}=2$

$x=10$

Example: If John invests $2300 in a savings account with a 9% interest rate compounded quarterly, how long will it take until John's account has a balance of $4150?

$A=P\left(1+\frac{r}{k}\right)^{kt}$

$4150=2300\left(1+\frac{0.09}{4}\right)^{4t}$

numerical solve

$t=6.63$

about 7 years.

Example: What interest rate compounded monthly is required for an $8500 investment to triple in 5 years?

$k=12$

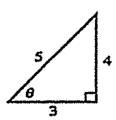
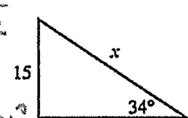
$25500=8500\left(1+\frac{r}{12}\right)^{12\cdot 5}$

numerical solve

$r\approx 0.22$

22%

— p3 - key.

22.Convert between Radian and degree (p325 #9-24). **Example: Convert to radians:** 60° $60 \cdot \frac{\pi}{180} = \frac{60}{180}\pi = \frac{1}{3}\pi = \frac{\pi}{3}$ **Example: Convert to degree:** $13\pi/20$ $\frac{13\pi}{20} \cdot \frac{180}{\pi} = 13 \cdot 6 = 78^\circ$	
23.Evaluate trig functions in a right triangle (p335#1-8, 49-58, p336#61,62, p347#7-12, 25-42) • **Example: Find all missing trig functions**  $\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{3}$ $\tan\theta = \frac{4}{3}$ $\cot\theta = \frac{3}{4}$ • **Example:** $\sin\theta = 3/7$  $3^2 + x^2 = 7^2$ $x^2 = 49 - 9$ $x^2 = 40$, $x = \sqrt{40} = 2\sqrt{10}$ $\cos = \frac{2\sqrt{10}}{7}$ $\tan = \frac{3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{20}$ $\cot\theta = \frac{2\sqrt{10}}{3}$ $\sec = \frac{7}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{7\sqrt{10}}{20}$ $\csc = 7/3$ • **Example: Solve for the missing variable**  $\sin 34^\circ = \frac{15}{x}$ $\frac{x}{15} = \frac{1}{\sin 34^\circ}$ $x = \frac{15}{\sin 34^\circ} = 26.8$	$\csc\theta = \frac{5}{4}$
24.Find the amplitude, period and frequency of sin and cos graphs from their equation (p357#13-16) **Example: Find amplitude, period, and frequency;** $y = 2\cos\frac{x}{3}$	S2

– p4-key