

Section 2.6: Graphs of Rational Functions.

Sample Problem:

Given the following Function:

$$f(x) = \frac{x^2 - 9}{x^2 + 4x + 3} \quad \text{Rational}$$

$$\frac{O}{K} \quad \frac{N}{O}$$

Find:

a) Domain

$$D = x^2 + 4x + 9 = 0 \\ (x+1)(x+3) = 0$$

$$\begin{array}{c} \xrightarrow{-3 \quad -1} \\ \frac{1}{4} \end{array} \begin{array}{c} 3 \\ 3 \\ 3 \end{array} \\ (-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$$

b) V.A $x = -1$, $x = -3$ hole

c) H.A $y = 1$ (same degree $N = D$) standard form

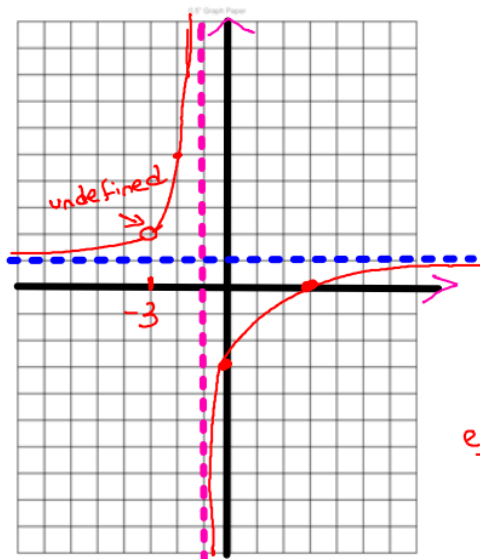
d) x-intercept $\rightarrow y = 0$ $(3, 0)$

e) y-intercept $\rightarrow x = 0$ (factored or standard) $\frac{0-9}{0+0+3} = -3$
 $(0, -3)$

f) Sketch the graph.

$$f(x) = \frac{x^2 - 9}{x^2 + 4x + 3} = \frac{(x-3)(x+3)}{(x+1)(x+3)} \quad \text{hole at } -3$$

$$\textcircled{y} = \frac{x^2 - 9}{x^2 + 4x + 3} \quad \begin{array}{l} x^2 - 9 = 0 \\ (x-3)(x+3) = 0 \\ x = 3 \quad x = -3 \text{ hole} \end{array}$$



V.A $x = -1$
 $x = -3$ hole^o

H.A $y = 1$

x-int $(3, 0)$

y-int $(0, -3)$

extra: $x = -2$

$$\frac{(-2)^2 - 9}{(-2)^2 + 4(-2) + 3}$$

$$= \frac{4 - 9}{4 - 8 + 3} = \frac{-5}{-1} = 5$$