Rational zero theorem (201 Green Box)

Example:

For a cubic polynomial, if none of the factors are given, and calculator is not allowed, then try the factors of the constant, plug them in $P(x)$, the one that makes $P(x)=0$ will be one of the polynomial roots, use it for synthetic $\div$ to factor $P(x)$ completely.

$P(x) = x^3 - 5x^2 + 2x - 10 = 0$

$\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10$

$P(5) = 5^3 - 5(5^2) + 2(5) - 10$

$= 125 - 125 + 10 - 10$

$P(5) = 0$

5 is one of the x-intercept/zeros of P(x)

$$\begin{array}{c|cccc} 5 & 1 & -5 & 2 & -10 \\ & \downarrow & 5 & 0 & 10 \\ \hline & 1x^2 & 0x & 2 & 0 \end{array}$$

R

constant

$P(x) = (x^2 + 2)(x - 5)$

$x^2 = -2$

$\boxed{x = \pm\sqrt{2}\, i}$

$x - 5 = 0$

$\boxed{x = 5}$

- Question factor P(x) completely $(x^2+2)(x-5)$
- Find real zeros $x = 5$
- Find all zeros of P(x) $x = 5$, $x = \sqrt{2}i$, $x = -\sqrt{2}i$

Example 2: $f(x) = x^3 + 3x^2 - 3x - 9$

$\pm 1 \quad \pm 3 \quad \pm 9$

$f(-3) = (-3)^3 + 3(-3)^2 - 3(-3) - 9$
$= -27 + 27 + 9 - 9$
$= 0$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -3 & -9 \\ & \downarrow & -3 & 0 & 9 \\ \hline & 1x^2 & 0x & -3 & 0 \end{array}$$

constant R

- Factor: $(x^2-3)(x+3)$
- Solve $x = -3$, $x^2 = 3$
  $x = \pm\sqrt{3}$.
- real zeros: $x = -3$, $x = \sqrt{3}$, $x = -\sqrt{3}$
- Rational zeros: $x = -3$
- Irrational zeros: $x = \sqrt{3}$, $x = -\sqrt{3}$.