

Special Cases

Left to right you factor
Right to left you distribute

Identity 1 $a^2 + 2ab + b^2 = (a + b)^2$

Identity 2 $a^2 - 2ab + b^2 = (a - b)^2$

Identity 3 $a^2 - b^2 = (a + b)(a - b)$

Identity 4 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Identity 5 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples

1) $(x+5)(x-5)$
 $(a+b)(a-b) = a^2 - b^2$
 $x^2 - 5^2 = x^2 - 25$

2) $(n-1)(n+1)$
 $(a-b)(a+b) = a^2 - b^2$
 $n^2 - 1^2 = n^2 - 1$

3) $(p-1)^2$
 $(a-b)^2 = a^2 - 2ab + b^2$
 $p^2 - 2 \cdot p \cdot 1 + 1^2$
 $= p^2 - 2p + 1$

4) $(x-3)(x+3)$
 $(a-b)(a+b) = a^2 - b^2$
 $x^2 - 3^2 = x^2 - 9$

5) $(x-4)^2$
 $(a-b)^2 = a^2 - 2ab + b^2$
 $= x^2 - 2 \cdot x \cdot 4 + 4^2$
 $= x^2 - 8x + 16$

6) $(n+3)^2$
 $(a+b)^2 = a^2 + 2ab + b^2$
 $(n+3)^2 = n^2 + 3 \cdot n \cdot 2 + 3^2$
 $= n^2 + 6n + 9$

Special Factoring Rules

Difference of Two Squares

Quadratics of the form $a^2 - b^2$ are called a difference of two squares and can be factored using the formula:

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1:

Factor each quadratic completely.

a.) $x^2 - 9 = (x-3)(x+3)$
 $a^2 - b^2 = (a-b)(a+b)$

b.) $x^2 - 16 = (x-4)(x+4)$
 $a^2 - b^2 = (a-b)(a+b)$

c.) $2x^2 - 8 = 2(x^2 - 4)$
 $= 2(x-2)(x+2)$

Since both terms divide evenly by 2, we factor out the 2:

$$2x^2 - 8 = 2(x^2 - 4)$$

The resulting quadratic is a difference of two squares, therefore we can factor further:

$$2(x^2 - 4) = 2(x+2)(x-2)$$

NOTE: a sum of two squares CANNOT be factored! It is considered to be PRIME.