$$f(x) = \frac{(x+2)(x-3)}{(x+2)(x-5)}$$

$x+2=0$ $x-5=0$
$x=-2$ $x=5$

- An **asymptote** is a line that a graph approaches without touching

- A **vertical asymptote** occurs in a rational function at any value of *x* for which the denominator is equal to 0, but for which the numerator is not equal to 0.

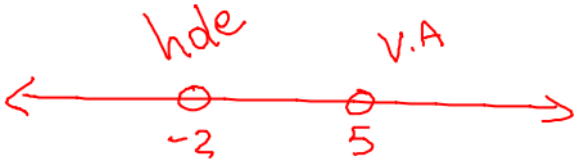
- When a value of *x* sets both the denominator **and** the numerator of a rational function equal to 0, there is a **hole** in the graph; that is, a single point at which the function has no value

Ex1

$$f(x) = \frac{(x+2)(x-3)}{(x+2)(x-5)}$$

$(x+2)(x-5) \neq 0$

$x \neq -2$ $x \neq 5$



*Examples*: Name the vertical asymptotes and holes in the graphs of the following equations:

1. $f(x) = \dfrac{(x-2)(x+4)}{(x-3)(x+3)(x+4)}$

hole at $x = -4$

V.A $x = -3$ , $x = 3$

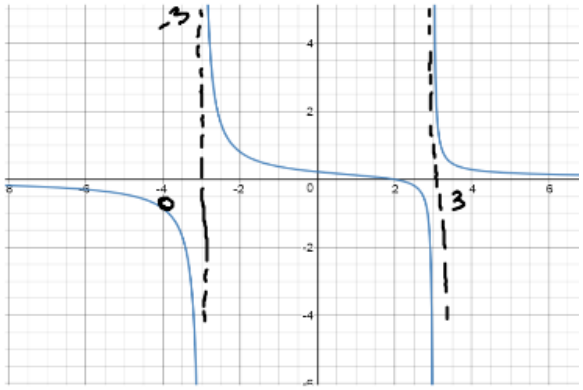
2. $f(x) = \dfrac{x^2(x+2)}{x^3(x-2)^2}$ ↙ v.A at $x = 2$

at $x = 0$ hole

3. $f(x) = \dfrac{(x-1)^2}{(x+1)(2x-3)(\frac{1}{2}x+6)}$

No holes.

$x = -1$ , $x = \frac{3}{2}$ , $x = -12$



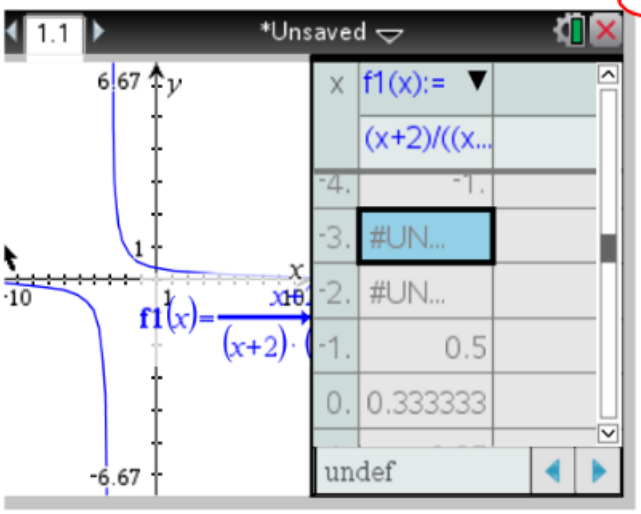
Find the domain of, vertical asymptotes and holes

Ex 2: $f(x) = \dfrac{x+2}{x^2+5x+6}$

$= \dfrac{x+2}{(x+2)(x+3)}$

hole at $x = -2$

$x = -3$ V.A



# Horizontal Asymptotes

- In order the find H.A look at the degree of the polynomials in the numerator and denominator

1. **If both polynomials are the same degree, divide the coefficients of the highest degree terms.**

$$f(x) = \frac{6x^{2} - 3x + 4}{2x^{2} - 8}$$ H.A: $y = 3$

2. **Example:**
Both polynomials are 2nd degree, so the asymptote is at ----------

**2. If the polynomial in the numerator is a lower degree than the denominator, the x-axis (y = 0) is the horizontal asymptote.**

$$\frac{x^{1} - 12}{2x^{3} + 5x - 3}$$ H.A x-axis

**3. If the polynomial in the numerator is a higher degree than the denominator, there is no horizontal asymptote.** There is a slant asymptote, which we will study in a later lesson.

$$f(x) = \frac{x^{2} - 9}{x + 10}$$ NO H.A

H.A (N ≷ D standard form)

- N = D: divide coefficient — $\frac{6x^{2} - 3x + 4}{2x^{2} - 8}$ — $y = \frac{6}{2} = 3$ H.A
- N < D: x-axis (y=0) H.A — $\frac{x - 12}{2x^{3} + 5x - 3}$
- N > D: NO H.A — $\frac{x^{2} - 9}{x + 10}$

H.A → Horizontal Asymptote
N → Numerator
D → Denominator

**Practice:** Find the horizontal asymptote of each rational function.

1) $f(x) = \dfrac{5x^3}{x^2 - 4x + 2}$ N > D No H.A

2) $f(x) = \dfrac{7x - 2}{x + 3}$ $y = 7$ H.A

3) $f(x) = \dfrac{3x^2 - x + 12}{2x^2 - 6x + 7}$ $y = \dfrac{3}{2}$

4) $f(x) = \dfrac{4x + 7}{6x^2 - 5}$ N < D x-axis ($y = 0$) is H.A

5) $f(x) = \dfrac{8x^2 - 5x + 1}{4x^2 - 3}$ $y = \dfrac{8}{4} = 2$