Continuous vs Discontinuous

- A function is continuous at a point, if the graph does not come apart at that point

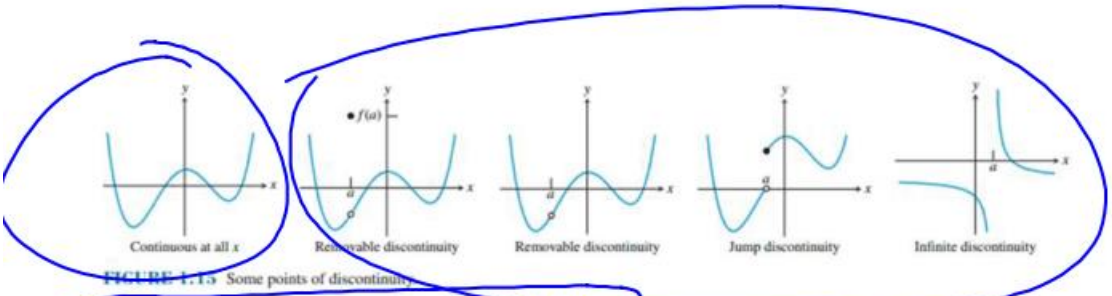

FIGURE 1.13 Some points of discontinuity

Figure 1.15 p.84

Tuesday 10/03

- Determine the points of discontinuity of the following functions and state whether at those points there is a vertical asymptote or a hole

- $f(x) = \frac{2}{x-4}$ Rational functions $x-4 \neq 0$, $x \neq 4$, V.A at $x=4$
- $g(x) = \frac{2}{x^2+1}$ $x^2+1 \neq 0$, $x^2 \neq -1$, No solution, No V.A
- $h(x) = \frac{(x+2)(x-4)}{(x-4)}$ $x-4 \neq 0$, $x \neq 4$, Hole at $x=4$

Horizontal Asymptotes → degree of N and D.

$f(x) = \frac{2x^3 + 7x - 2}{5x^3 + 6x^2 + 7}$ same degree H.A at $y = \frac{2}{5}$

$g(x) = \frac{2x^3 + 7x - 2}{5x^4 + 6x^2 + 7}$ degree N < degree D, H.A is at $y = 0$ (x-axis)

$h(x) = \frac{2x^3 + 7x - 2}{5x^2 + 6x + 7}$ degree N > degree D, No H.A

The method used to find the horizontal asymptote changes depending on how the degrees of the polynomials in the numerator and denominator of the function compare.

- **If both polynomials are the same degree, divide the coefficients of the highest degree terms.**

  Example:

  $f(x) = \frac{6x^2 - 3x + 4}{2x^2 - 8}$

  Both polynomials are 2nd degree, so the asymptote is at $y = \frac{6}{2}$ or $y = 3$
- **If the polynomial in the numerator is a lower degree than the denominator, the x-axis (y = 0) is the horizontal asymptote.**
- **If the polynomial in the numerator is a higher degree than the denominator, there is no horizontal asymptote.** There is a slant asymptote, which we will study in a later lesson.

Practice: Find the horizontal asymptote of each rational function.

1) $f(x) = \frac{5x^3}{x^2 - 4x + 2}$ Degree N > Degree D
No H.A

2) $f(x) = \frac{7x - 2}{1x + 3}$ same degree H.A at $y = 7$

3) $f(x) = \frac{3x^2 - x + 12}{2x^2 - 6x + 7}$ same degree H.A at $y = \frac{3}{2}$

4) $f(x) = \frac{4x + 7}{6x^2 - 5}$ degree N < degree D
H.A $y = 0$

5) $f(x) = \frac{8x^2 - 5x + 1}{4x^2 - 3}$ Same degree $y = 2$

Identify any H.A (or) V.A of the graph of:

1) $f(x) = \frac{x-5}{3x^2+1}$

V.A $3x^2 + 1 = 0$
$3x^2 = -1$ No sol)
No V.A
degree N < degree D
H.A at $y = 0$

2) $g(x) = \frac{2x^2}{4 - 1x^2}$

V.A at $x = -2$ and $x = 2$
same degree
H.A $y = \frac{2}{-1} = -2$

3) $h(x) = \frac{\frac{2x^2}{(2-x)(2+x)}}{x^2 - x - 2}$

H.A at $y = 0$ degree N < degree D

V.A $x^2 - x - 2 \neq 0$
$(x-2)(x+1) \neq 0$
V.A at $x = 2$ and $x = -1$

-2 | -2 | 1 | -1