'26/20 Progress check MCQ

$y = f(x)$ is the particular solution to $\frac{dy}{dx} = \frac{e^x - 1}{e^y}$ $f(1) = 0$
$f(-2) = ?$

separate variables $e^y dy = (e^x - 1)dx$

integrate both sides $\int e^y dy = \int (e^x - 1)dx$

$$e^y = e^x - x + C$$

use initial condition to solve for c $e^0 = e^1 - 1 + C$

$$1 = e - 1 + C$$

$$C = 2 - e$$

particular solution $e^y = e^x - x + 2 - e$

$$y = \ln(e^x - x + 2 - e)$$

Solve for $f(-2)$ $y(-2) = \ln(e^{-2} + 2 + 2 - e) = 0.349$ B

2. rate of change of volume with respect to time t leaking is proportional to the cube of volume

$\frac{dV}{dt} = - K \cdot V^3$

$\frac{dV}{dt} = -K \cdot V^3$ B

3. The rate of change of T with respect to time is proportional to the difference between egg's temp (°F) and the 75°F

$\frac{dT}{dt} = K(T - 75)$ D

4. N = total # of students

S = # of students who have seen the program

N - S = # of students who have not seen the program

$$\frac{dS}{dt} = kS(N-S)$$

C

5. $xy' - 3y = 6$

a) $y = x^2 + 2$ $\quad y' = 2x$

plug i $\quad x(2x) - 3(x^2+2) = 6$

$2x^2 - 3x^2 - 6 = 6$

$-x^2 - 6 = 6$ False

b) $y = 3x^2 - 2$ $\quad y' = 6x$

plug in $\quad x(6x) - 3(3x^2 - 2) = 6$

$6x^2 - 9x^2 + 6 = 6$

$-3x^2 + 6 = 6$ False

c) $y = 5x^3 - 2$ $\quad y' = 15x^2$

plug in $\quad x(15x^2) - 3(5x^3 - 2) = 6$

$15x^3 - 15x^3 + 6 = 6$

$6 = 6$ True

C

6. $4y + y'' = 0$

I $\quad y = 3\sin 2x$ $\quad y' = 3\cos 2x \cdot 2 = 6\cos 2x$

$y'' = -6\sin 2x \cdot 2 = -12\sin 2x$

plug in $\quad 4 \cdot 3\sin 2x - 12\sin 2x = 0$

$12\sin 2x - 12\sin 2x = 0$ True

II $\quad y = 5\cos 2x$ $\quad y' = -10\sin 2x$ $\quad y'' = -20\cos 2x$

plug in $\quad 4 \cdot 5\cos 2x - 20\cos 2x = 0$

$20\cos 2x - 20\cos 2x = 0$ True

III $\quad y = e^{2x}$

$y' = 2e^{2x}$ $\quad y'' = 4e^{2x}$

plug in $\quad 4 \cdot e^{2x} + 4e^{2x} = 0$

False

D

7. $y''' - 4y' = 0$

A. $y = 10 \quad y' = 0 \quad y'' = 0 \quad y''' = 0$

$0 - 4 \cdot 0 = 0$ True

B. $y = 4e^{-2x} \quad y' = -8e^{-2x} \quad y'' = 16e^{-2x} \quad y''' = -32e^{-2x}$

$-32e^{-2x} - 4 \cdot (-8e^{-2x}) = 0$

$-32e^{-2x} + 32e^{-2x} = 0$ true

C. $y = 3\sin 2x \quad y' = 6\cos 2x \quad y'' = -12\sin 2x \quad y''' = -24\cos 2x$

$-24\cos 2x - 4(6\cos 2x) = 0$ C

$-24\cos 2x - 24\cos 2x = 0$ False

8. slopes are zero along x and y axis so ~~A~~, ~~A~~

In Q_3 slopes are $(-)$ -> B

Slope = 1 for $y = x$ ~~A~~, ~~B~~, ~~C~~

9. D $\frac{dy}{dx} = x - y + 1$

10. $\frac{dy}{dx} = y^3 - x$ B

point	$\frac{dy}{dx}$
$(\frac{1}{2}, \frac{1}{2})$	$\frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$
$(-\frac{1}{2}, \frac{1}{2})$	$\frac{1}{8} + \frac{1}{2} = \frac{5}{8}$
$(-\frac{1}{2}, \frac{1}{2})$	$-\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$
$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{8} - \frac{1}{2} = -\frac{5}{8}$

-> ~~A~~ · C (at this point slope is +)

if $y = \frac{1}{8}$ and $-1 < x < 1$ slope is + and decreasing in Q_2 and is − and increasing in Q_1 ~~D~~

11. $\frac{dy}{dx} = \frac{\cos 8x}{\cos 4y}$

$\int \cos 4y \, dy = \int \cos 8x \, dx$

$-\frac{1}{4} \sin 4y = -\frac{1}{8} \sin 8x + C_1 \quad \cdot 8$

C

$-2 \sin 4y = -\sin 8x + 8C_1 \qquad -8 \cdot C_1 = C$

$2\sin 4y - \sin 8x = C$

12. $\dfrac{dy}{dx} = \dfrac{3x^2-1}{2y}$ $f(1)=4$ $y=f(x)=?$

Separate variables and integrate $\int 2y\,dy = \int 3x^2-1\,dx$

General solution $y^2 = x^3 - x + C$ use $f(1)=4$

$4^2 = 1^3 - 1 + C$

$C = 16$

particular solution $y^2 = x^3 - x + 16$

$y = \sqrt{x^2 - x + 16}$ B

13. $\dfrac{dy}{dx} = \dfrac{2x-1}{y^2}$ $y(0)=3$ $y=f(x)=?$

Separate variables and integrate $\int y^2\,dy = \int (2x-1)\,dx$

$\dfrac{y^3}{3} = x^2 - x + C$

General solution $y^3 = 3x^2 - 3x + 3C$

use $y(0)=3$ to solve for C $27 = 3C$ $C = 9$

Particular solution $y = \sqrt[3]{3x^2 - 3x + 27}$ D

14. $\dfrac{dP}{dt} + P = 10$ $P(0)=4$ $P=?$

Separate variables and integrate $\dfrac{dP}{dt} = 10 - P$ $\int \dfrac{dP}{10-P} = \int dt$ $-\ln|10-P| = t + C$

$\ln|10-P| = -t - C$

General solution

use $P(0)=4$ to find C

$\ln|10-4| = -C$ $C = -\ln 6$

Particular solution $\ln|10-P| = -t + \ln 6$

$e^{\ln|10-P|} = e^{-t+\ln 6} = e^{-t} \cdot e^{\ln 6}$

$|10-P| = 6e^{-t}$

$10 - P = \pm 6e^{-t}$

$P = 10 \pm 6e^{-t}$ for $P(0) = 4$ means use only "$-$"

$P = 10 - 6e^{-t}$ C

15. $\frac{dy}{dx} = \sin x^2$ $y(\sqrt{\pi}) = 4$ $y(x) = ?$

Separate variables and integrate

$\int dy = \int \sin x^2 \, dx$

$y = \int \underline{\sin x^2 \, dx}$ we do not know how to solve this

$\cancel{A}$ $\cancel{B}$ $\cancel{C}$ because initial condition is $y(\underline{\sqrt{\pi}}) = 4$

use D to check the solution

$y = 4 + \int_{\sqrt{\pi}}^{x} \sin(t^2)\, dt$

$\frac{dy}{dx} = y' = \frac{d}{dx} \int_{\sqrt{\pi}}^{x} \sin(t^2) dt = \sin x^2$
(using Fundamental th. of calc. Part 1)

Plug in $\sin x^2 = \sin x^2$ true D

16. The rate of change of the car's (value) is proportional to the car's value

$\frac{dV}{dt} = K \cdot V$ B

V = value of the car

17. $\frac{dV}{dt} = kV$ $V(0) = 400 \text{ km}^3$
when $V = 300 \text{ km}^3$ $\frac{dV}{dt} = -15 \text{ km}^3/\text{year}$ ↳ decreasing

$V(t) = ?$

when $V = 300 \text{ km}^3$ $\frac{dV}{dt} = -15$ $\Rightarrow$ $-15 = k \cdot 300$

$k = \frac{15}{300} = -\frac{1}{20}$

$$\frac{dV}{dt} = -\frac{1}{20} V$$

Separate variables and integrate $\int \frac{dV}{V} = \int -\frac{1}{20} \, dt$

$$\ln|V| = -\frac{1}{20}t + c$$

$$e^{\ln|V|} = e^{-\frac{1}{20}t + c}$$

$$|V| = e^{-1/20 t + c}$$

use initial condition $V(0) = 400$ to find C

$$400 = e^{C}$$

$$C = \ln 400$$

particular solution $V = e^{-1/20 t} \cdot e^{\ln 400}$

$V = 400 e^{-\frac{1}{20}t}$ C

18. The amount of bacteria $= B$
increases at a rate (relative to time) proportional to the amount present

$$\frac{dB}{dt} = kB$$

$B(0) = 10$
$B(2) = 30$
$B(6) = ?$

Separate variables and integrate

$$\int \frac{dB}{B} = \int K\,dt$$

$$\ln|B| = K \cdot t + C_1$$

$$e^{\ln|B|} = e^{Kt + C_1}$$

$$B = e^{Kt + C_1} = e^{Kt} \cdot e^{C_1} = C e^{Kt}$$

use initial condition to solve for C

$B(0) = 10$

$$10 = C \cdot e^0$$

$$C = 10 \quad \Rightarrow \quad B = 10 e^{Kt}$$

use given condition $B(2) = 30$ to solve for K

$$30 = 10 e^{K \cdot 2}$$

$$3 = e^{2K} \qquad 2K = \ln 3 \qquad K = \frac{\ln 3}{2}$$

particular solution $B = 10 \cdot e^{\frac{\ln 3}{2} t}$

$$B(6) = 30\ e^{\frac{1}{2} \cdot 0} = 270 \text{ grams}$$

C