3/24/20

Unit 7 Progress check FRQ part A.

$\frac{dA}{dt} = 6 - 0.02A$ at $t = 10$ min

$A = 50$ lb

a) tangent line at $t = 10$ min

$y - y_1 = m(x - x_1)$

$y_1 = 50$

$x_1 = 10$

$m = \text{slope} = \frac{dA}{dt} = 6 - 0.02 \cdot 50$

$m = 5$

$y - 50 = 5(t - 10)$

Use this equation to approximate $A = y$ at $t = 12$ min

$y - 50 = 5(12 - 10)$

$y - 50 = 10$

$\boxed{A = y \approx 60 \text{ lb}}$

b) Show that $A_{(t)} = 300 - 250e^{0.2 - 0.02t}$ is a solution

$\frac{dA}{dt} = -250 \cdot e^{0.2-0.02t}(-0.02)$

$\frac{dA}{dt} = 5 \cdot e^{0.02-0.02t}$

Plug in into the given eq.

$5 \cdot e^{0.02-0.02t} = 6 - 0.02 \cdot (300 - 250 \cdot e^{0.2-0.02t})$

$5 \cdot e^{0.02-0.02t} = \cancel{6} - \cancel{6} + 5 \cdot e^{0.2-0.02t}$

$5 \cdot e^{0.02-0.02t} = 5 \cdot e^{0.2-0.02t}$ true ✓

e) $\frac{dh}{dt} = -k\sqrt{h}$ $\quad (0,16)$ $\quad$ find $h(t)$

$(30,4)$

Separate variables

$$\frac{dh}{\sqrt{h}} = -k\,dt$$

Integrate both sides to find general solution

$$\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$$

$$2h^{\frac{1}{2}} = -k \cdot t + C$$

Use given conditions to find k and C

$(0,16)$ $\quad 2 \cdot 16^{1/2} = -k \cdot 0 + C$

$$8 = C$$

$(30,4)$: $\quad 2 \cdot 4^{1/2} = -k \cdot 30 + 8$

$$4 = -30k + 8$$

$$k = \frac{4}{30} = \frac{2}{15}$$

Use k and C to write particular solution

$$2h^{1/2} = -\frac{2}{15}t + 8$$

$$h^{1/2} = -\frac{1}{15}t + 4$$

$$\boxed{h(t) = \left(-\frac{1}{15}t + 4\right)^2}$$

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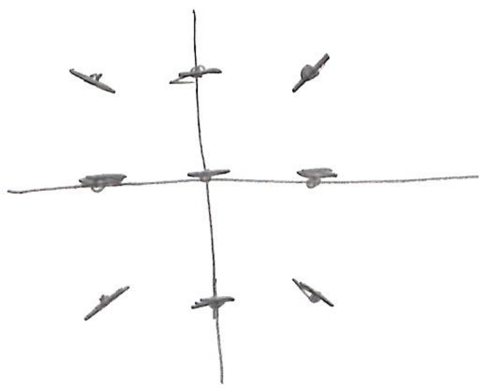
Unit 7 Progress Check FRQ part B

1. $\frac{dy}{dx} = xy^3$

a)



$(0,0)$ $\frac{dy}{dx} = 0$

$(0,1)$ $\frac{dy}{dx} = 0$

$(0,-1)$ $\frac{dy}{dx} = 0$

$(-1,0)$ $\frac{dy}{dx} = 0$

$(1,0)$ $\frac{dy}{dx} = 0$

$(-1,1)$ $\frac{dy}{dx} = -1$

$(-1,-1)$ $\frac{dy}{dx} = 1$

$(1,1)$ $\frac{dy}{dx} = 1$

$(1,-1)$ $\frac{dy}{dx} = -1$

b) $\frac{d^2y}{dx^2} = y^3 + x \cdot 3y^2 \cdot \frac{dy}{dx}$ substitute $\frac{dy}{dx} = xy^3$

$\frac{d^2y}{dx^2} = y^3 + 3xy^2(x \cdot y^3)$

$\frac{d^2y}{dx^2} = y^3 + 3x^2y^5$

In Quadrant IV $x>0$ and $y<0$, therefore $\frac{d^2y}{dx^2} < 0$

$y^3 < 0$

$3x^2y^5 < 0$

If $\frac{d^2y}{dx^2} < 0$ than $y(x)$ is going concave down.

c) $y = f(x) = ?$ $f(2) = -1$

$\frac{dy}{dx} = xy^3$

Separate variables $\frac{dy}{y^3} = x\,dx$

Integrate both side to find general solution $\int \frac{dy}{y^3} = \int x\,dx$ $-\frac{y^{-2}}{2} = \frac{x^2}{2} + C$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

use the initial condition to find constant C:

$f(2) = -1 \qquad y = -1$
$\qquad\qquad\qquad x = 2$

$$-\frac{1}{2(-1)^2} = \frac{2^2}{2} + C$$

$$-\frac{1}{2} = 2 + C$$

$$C = -\frac{1}{2} - 2$$

$$C = -\frac{5}{2}$$

use constant C to write particular solution

$$-2 \cdot \left(-\frac{1}{2y^2}\right) = \left(\frac{x^2}{2} - \frac{5}{2}\right)(-2)$$

$$\frac{1}{y^2} = -x^2 + 5$$

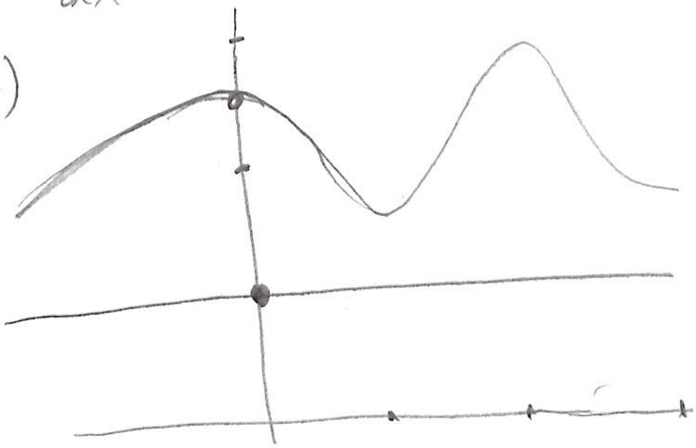
$$y^2 = \frac{1}{-x^2+5}$$

$$\boxed{y = -\frac{1}{\sqrt{-x^2+5}}}$$

because the solutions includes the initial condition (2, -1)

2. $\frac{dy}{dx} = (y-1)(x+1)\cos(x^2+2x+\pi)$

a)



b) Eq of tangent line at $(0, \frac{5}{2})$

$y - y_1 = m(x - x_1)$

$m = \text{slope} = \frac{dy}{dx}$ at $(0, \frac{5}{2})$

$m = (\frac{5}{2} - 1)(0+1) \cdot \cos(0^2 + 2 \cdot 0 + \pi)$

$m = \frac{3}{2} \cdot 1 \cdot \cos \pi$

$m = -\frac{3}{2}$

Eq of tangent line $y - \frac{5}{2} = -\frac{3}{2}(x - 0)$

or $y = -\frac{3}{2}x + \frac{5}{2}$

c) particular solution $y = f(x) = ?$ if $f(0) = \frac{5}{2}$

$\frac{dy}{dx} = (y-1)(x+1)\cos(x^2+2x+\pi)$

Separate the variables $\frac{dy}{y-1} = (x+1)\cos(x^2+2x+\pi)\,dx$

Integrate both sides to find general solution $\int \frac{dy}{y-1} = \int (x+1)\cos(x^2+2x+\pi)\,dx$ $u = x^2+2x+\pi$, $du = 2x+2$

$\ln|y-1| = \frac{1}{2}\int \cos u \, du$

$\ln|y-1| = \frac{1}{2}\sin u + C$

$\ln|y-1| = \frac{1}{2}\sin(x^2+2x+\pi) + C$

$e^{\ln|y-1|} = e^{\frac{1}{2}\sin(x^2+2x+\pi) + C}$

$|y-1| = e^{\frac{1}{2}\sin(x^2+2x+\pi)} \cdot e^{c}$ if $e^c = c_1$

$y = 1 \pm C_1 e^{\frac{1}{2}\sin(x^2+2x+\pi)}$

use initial condition $f(0) = \frac{5}{2}$ to find c_1

$$\frac{5}{2} = 1 \pm c_1 e^{\frac{1}{2}\sin(0^2+2\cdot 0+\pi)}$$

$$\frac{5}{2} = 1 \pm c_1 e^0$$

$$\frac{5}{2} = 1 \pm c_1$$

$$c_1 = \pm \frac{3}{2}$$

Use c_1 to write particular solution

$$\boxed{y = 1 + \frac{3}{2} e^{\frac{1}{2}\sin(x^2+2x+\frac{\pi}{2})}}$$

only the "+" because the initial condition $(0, \frac{5}{2})$

$y > 1$