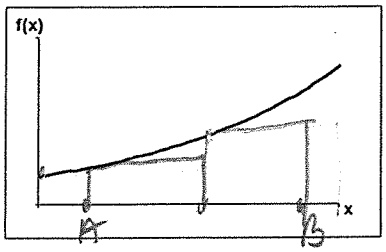
Key

Chapter 5 Review

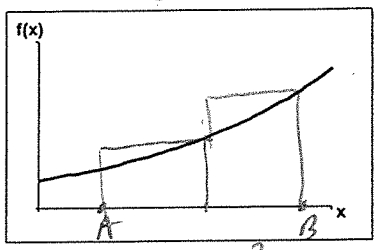
1. Assume we are trying to estimate the value of $\int_A^B f(x)dx$. Illustrate the indicated rule with $n = 2$ in each diagram. Include a formula for each estimate and a general formula for the rule.

Left hand rule



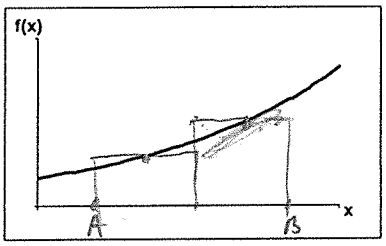
$$\int_A^B f(x)dx \approx \sum_{k=1}^{2} \frac{B-A}{2} \cdot f\left(A + \frac{B-A}{2}\cdot(k-1)\right)$$

Right hand rule



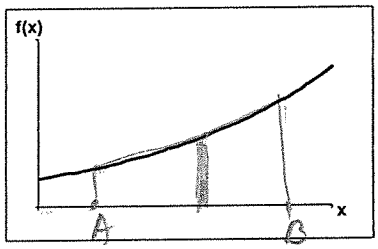
$$\int_A^B f(x)dx = \sum_{k=1}^{2} \frac{B-A}{2} \cdot f\left(A + \frac{B-A}{2}\cdot k\right)$$

Midpoint rule



$$\int_A^B f(x)dx \approx \sum_{k=1}^{2} \frac{B-A}{2} f\left(A + \frac{B-A}{2}\left(k-\tfrac{1}{2}\right)\right)$$

Trapezoid rule



$$\int_A^B f(x)dx \approx \frac{B-A}{2}\left(f(A) + 2f\left(\frac{B+A}{2}\right) + f(B)\right)$$

2. Complete the table using the words "overestimate" or "underestimate".

	Shape of Graph			
Rule	Increasing Concave Up	Increasing Concave Down	Decreasing Concave Up	Decreasing Concave Down
Left hand	under	under	over	over
Right hand	over	over	under	under
Midpoint	under	over	under	over
Trapezoid	over	under	over	under

3. Suppose we estimate $\int_A^B f(x)dx$ using our rules with the same number of subdivisions, n but only record three of our estimates: $Right(n) = 1.8569$ $Mid(n) = 2.3481$ $Trap(n) = 2.1627$.
If $f(x)$ is monotone and does not have any inflection points in the interval $[A, B]$,

A. Is $f(x)$ increasing or decreasing? B. Is $f(x)$ concave up or down?

C. Estimate the value of $Left(n)$ and ~~$Simp(n)$~~

2.4685

4. Given: $\int_3^1 f(x)dx = -3$ & $\int_3^5 f(x)dx = 7$
Find: $\int_1^5 2f(x)dx$ $= 2\left(\int_1^3 f(x)dx + \int_3^5 f(x)dx\right) = 2(3+7) = 20$

5. Given: $\int_0^9 f(x)dx = 12$ & $\int_9^3 f(x)dx = 5$
Find: $\int_0^3 f(x)dx$ $= \int_0^9 f(x)dx - \int_3^9 f(x)dx = 12 - (-5) = 17$

6. Change the following integral notations into left Riemann sum notations

a. $\int_1^2 \frac{x^2+3}{x}dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \cdot f\left(1+\frac{1}{n}(k-1)\right)$ where $f(x) = \frac{x^2+3}{x}$

b. $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$ $= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{4n} \cdot f\left(0+\frac{\pi}{4n}(k-1)\right)$ where $f(x) = \sec^2 x$

7. Evaluate: $\frac{d}{dx}\int_3^{x^3} \tan(2x)\,dx$ $= \frac{d}{dx}\left(F(2x^3) - F(3)\right)$
$= 3x^2 \cdot \frac{d}{dx}F(x^3) = 3x^2 \cdot \tan(2x^3)$

8. Evaluate: $\frac{d}{dx}\int_5^{\sin(3x)} (x^2-5)\,dx$ $= \frac{d}{dx}\left(F(\sin(3x)) - F(5)\right)$
$= 3\cdot\cos(3x)\frac{d}{dx}F(\sin 3x) = 3\cos(3x)\cdot\left((\sin 3x)^2 - 5\right)$

9. Evaluate: $\frac{d}{dx}\int_{2x^2}^3 (x^2-5x+1)\,dx$ $= \frac{d}{dx}\left(F(3) - F(2x^2)\right) = -4x \cdot \frac{d}{dx}F(2x^2)$
$= -4x\cdot(4x^4 - 10x + 1)$

From your book page 316 and 317 # 12, 38-42

page 316-317

12. a) $\int_0^{10} x^3 dx$

b) $\int_0^{10} x \sin x \, dx$

c) $\int_0^{10} x(3x-2)^2 dx$

d) $\int_0^{10} \frac{1}{1+x^2} dx$

e) $\int_0^{10} \pi \left(9 - \sin^2 \frac{\pi x}{10}\right) dx$

38. a) $y_{avg} = \frac{4}{3}$

b) $y_{avg} = \frac{2}{3} a^{3/2}$

39. $\frac{dy}{dx} = \sqrt{2+\cos^3 x}$

40. $\frac{dy}{dx} = 14x \sqrt{2+\cos^3(7x^2)}$

41. $\frac{dy}{dx} = \frac{-6}{3+x^4}$

42. $\frac{2}{4x^2+1} - \frac{1}{x^2+1}$