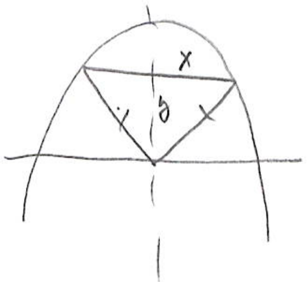
1. **An isosceles triangle has its vertex at the origin and its base parallel to the x-axis on the curve $y = 27 - x^2$. Find the largest possible area the triangle can have?**



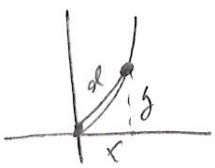
$A = 2 \cdot \frac{1}{2} x y = x \cdot y$; $y = 27 - x^2$; $A = x(27 - x^2) = 27x - x^3$

A_{max} if $A' = 0$ $\quad A' = 27 - 3x^2 = 0 \quad x = \pm 3$

A': -3: $- \; 0 \; +$, 3: $0 \; -$ max

A_{max} when $x = 3$ $\quad \boxed{A_{max} = 54 u^2}$

2. **A point moves smoothly along the curve $y = x^{3/2}$ in the first quadrant in such a way that the straight line distance from the origin increases at the constant rate of 11 units/sec. Find $\frac{dx}{dt}$ when $x = 3$.**



$d^2 = x^2 + y^2$; $(d^2 = x^2 + x^3)'$

$w = x^{3/2}$

$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 3x^2\frac{dx}{dt} \qquad 2d\frac{dd}{dt} = \frac{dx}{dt}(2x + 3x^2)$

$\frac{dx}{dt} = \frac{2d\frac{dd}{dt}}{2x + 3x^2}$ $\qquad$ when $x = 3$ $\quad y = 3^{3/2} = 3\sqrt{3}$ $\quad d = \sqrt{3^2 + 3^3}$ $\quad d = 6$

$\boxed{\frac{dx}{dt} = \frac{2 \cdot 6 \cdot 11}{2 \cdot 3 + 3 \cdot 3^2} = 4 \text{ units/sec}}$

3. **Gas is escaping from a spherical balloon at the rate of 2 ft^3/min. How fast is the surface area changing when the radius is 12 ft?**

$(V = \frac{4}{3}\pi R^3)' \quad V' = -2 ft^3/min \qquad (S.A = 4\pi R^2)'$

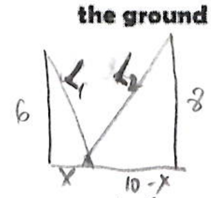
$V' = 4\pi R^2 R' \qquad (S.A)' = 8\pi R R'$

$R' = \frac{V'}{4\pi R^2}$ when $R = 12$ $R' = \frac{-2}{4\pi 12^2} = \frac{-1}{288\pi}$ $\qquad$ when $R = 12$

$(SA)' = 8\pi \cdot 12 \cdot \frac{-1}{288\pi} = -\frac{1}{3} ft^2/min$

$\boxed{\text{S.A is decreasing at a rate of } 1/3\ ft^2/min}$

4. **Two vertical poles of length 6 feet and 8 feet stand on level ground, with their bases 10 feet apart. Approximate the minimal length of a cable that can reach from the top of one pole to some point on the ground between the poles and then to the top of the other pole.** 2



$L = L_1 + L_2 = \sqrt{36 + x^2} + \sqrt{x^2 - 20x + 164}$

$L_1 = \sqrt{6^2 + x^2}$

$L_2 = \sqrt{(10-x)^2 + 8^2}$

$L \to min$ if $L' = 0$

$L' = \frac{x}{\sqrt{36 + x^2}} + \frac{x - 10}{\sqrt{x^2 - 20x + 164}} = 0$

$\frac{x}{\sqrt{36 + x^2}} = \frac{10 - x}{\sqrt{x^2 - 20x + 164}}$

$\frac{x^2}{36 + x^2} = \frac{x^2 - 20x + 100}{x^2 - 20x + 164}$

$\cancel{x^4} - \cancel{20x^3} + 164x^2 = 36x^2 - 720x + 3600 + \cancel{x^4} - \cancel{20x^3} + 100x^2$

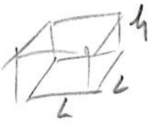
$28x^2 + 720x - 3600 = 0 \quad x = -30 \quad (x = 4.28)$

x: DNA -30, 4.28

L': $+ \; 0 \; - \; 0 \; +$ min

When $x = 4.28$ $\boxed{L = 17.2 ft}$

5. **A box with a square base is to be constructed to hold 64 cubic inches. What are the dimensions of such a box to minimize surface area?**



$V = L^2 \cdot h = 64 \quad h = \frac{64}{L^2}$

$S.A = 2L^2 + 4L \cdot h = 2L^2 + \frac{256}{L}$

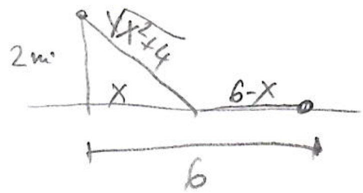
$S.A \to min$ where $S.A' = 0$

$(S.A)' = 4L - \frac{256}{L^2} = 0 \quad 4L = \frac{256}{L^2}$

$4L^3 = 256 \quad L = 4 \Rightarrow h = 4$

To minimize the S.A the box has to be a cube with side 4in

6. **Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?**



$$t = \frac{\sqrt{x^2+4}}{2} + \frac{6-x}{5}$$

$t = \min$ if $t' = 0$

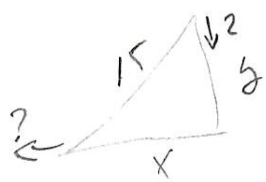
$$t' = \frac{2x}{4\sqrt{x^2+4}} - \frac{1}{5} = 0$$

$\frac{x}{\sqrt{x^2+4}} = \frac{1}{5}$ $5x = 2\sqrt{x^2+4}$ $25x^2 = 4x^2+16$ $21x^2 = 16$ $x = \pm 0.873$

t': $+$ 0 $-$ 0 $+$ at -0.873, 0.873 → min

to minimize time Jane should land her boat 0.873 mi from the point nearest to her initial position

7. **A 15 ft ladder is sliding down a building at a constant rate of 2 feet/min. How fast is the base of the ladder moving away from the building when the base of the ladder is 9 ft from the building?**



$(15^2 = x^2 + y^2)'$

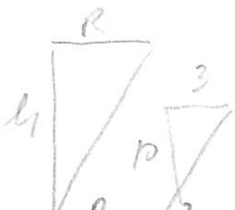
$0 = 2xx' + 2yy'$

$x' = \frac{-yy'}{x}$

when $x = 9$ $y = \sqrt{15^2 - 9^2} = 12$

$x' = \frac{-12 \cdot -2}{9} = \frac{8}{3}$ feet/min

8. **Water is being withdrawn from a conical reservoir 3 feet in radius and 10 feet deep at a rate of $4\ ft^3/\min$. How fast is the surface falling (depth of water decreasing) when the depth of the water is 6 ft.? How fast is the area of the surface decreasing at that instant?**



$V' = 4\ ft^3/min$

$-h' = ?$ when $h = 6ft$

$-A' = ?$ when $h = 6ft$

$\frac{R}{h} = \frac{3}{10} \Rightarrow R = \frac{3h}{10}$ $V = \frac{1}{3}\pi R^2 \cdot h$ $V = \frac{1}{3}\pi \frac{9h^2}{100} h$

$\left(V = \frac{3\pi}{100} h^3\right)'$

$V' = \frac{9\pi}{100} h^2 \cdot h'$

$h' = \frac{100 V'}{9\pi h^2}$

when $h = 6$ $h' = \frac{100 \cdot 4}{9\pi \cdot 36} = 0.393\ ft/min$

$A = \pi R^2 = \pi \cdot \frac{9h^2}{100}$

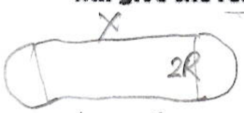
$\left(A = \frac{9\pi}{100} h^2\right)'$

$A' = \frac{9\pi}{50} \cdot h h'$

when $h = 6$ $A' = \frac{9\pi}{50} \cdot 6 \cdot (0.393)$

$A' = 1.333\ ft^2/min$

9. **An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the two ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?**



$2x + 2\pi R = 400$ $x = \frac{400 - 2\pi R}{2} = 200 - \pi R$ $A_\square = (200 - \pi R) \cdot 2R$

$A_\square = x \cdot 2R$

$(A_\square = 400R - 2\pi R^2)'$ $A_\square = \max$ when $A'_\square = 0$

$A'_\square = 400 - 4\pi R = 0$ $R = \frac{400}{4\pi} = \boxed{\frac{100}{\pi} m}$ $\Rightarrow x = 200 - \pi \cdot \frac{100}{\pi}$ $\boxed{x = 100\ m}$

When $R = \frac{100}{\pi} m$ and $x = 100\ m$ $A_\square$ is the largest

R: $\frac{100}{\pi}$; A': $+ +$ 0 $- -$ max

10. **In a rectangle, the length is increasing at a constant rate of 3.02 cm/sec, while the width is decreasing at a constant rate of 0.62 cm/sec. At the time the length is 2 cm and the width is 0.4 cm, what is the rate of change of the area?**

$L' = 3.02\ cm/sec$

$w' = -0.62\ cm/sec$

$A' = ?$

$(A = L \cdot w)'$

$A' = Lw' + L'w$

when $L = 2$, $w = 0.4$ $A' = 2 \cdot (-0.62) + (3.02) \cdot 0.4$

$\boxed{A' = -0.032\ cm^2/sec}$