AP Calculus AB Winter Break Packet

Name_____

Instructions: There are 20 problems in this packet. To get credit, you must have work shown in the box. If there is no work to show, then write an explanation. Not understanding a problem completely is NOT an excuse to leave the problem blank. There are hints on the back. You have dozens of pages of notes and assignments from this class, plus untold troves of information on the internet, including at Khan Academy and Youtube. ***It is due the first day after break at the beginning of the period.***

1.) The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the position is 10 when $t = 1$, then which of the following could be the function for position $x(t) =$?

(A) $9t^2 + 1$ (B) $3t^2 - 2t + 4$ (C) $t^3 - t^2 + 4t + 6$ (D) $t^3 - t^2 + 9t - 20$ (E) $36t^3 - 4t^2 - 8t + 5$

2.) At what value of x does the graph $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

3.) If $f(x) = (x-1)^2 \sin(x)$, then $f'(0) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

4.) Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

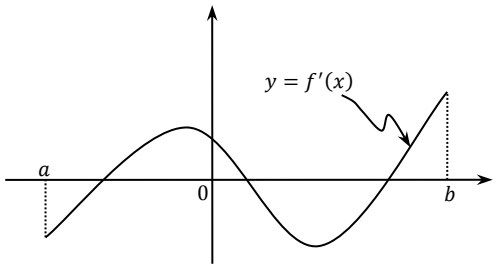
a.) Find the slope of the graph of f at the point where $x = 1$ and use it to write the tangent line to the graph.

b.) Use your answer from part (a) to approximate $f(1.2)$.

5.) If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

6.) The graph of f', the derivative of f, is shown in the figure below. Which of the following describes all relative extrema of f on the open interval (a, b)?



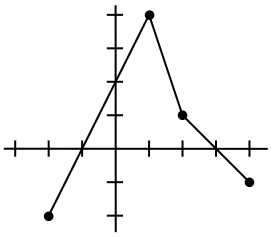
(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Two relative maxima and two relative minima

7.) Let f be the function defined by $f(x) = \begin{cases} x^3, & x \leq 0 \\ x, & x > 0 \end{cases}$. Which of the following statements about f is true?

(A) f is discontinuous at 0 (B) f has a relative maximum (C) $f'(x) > 0$ for all $x \neq 0$

(D) f is concave down for its entire domain (E) f is not differentiable at $x = 0$

8.) The graph of the function $f(t)$, consisting of three line segments, is shown to the right.



a.) If $g(x) = \int_1^x f(t)dt$, then find $g(4)$ and $g(-1)$.

b.) If $f(t)$ represents the derivative of a function, determine where the original function has a minimum value. Justify your answer.

c.) Again $f(t)$ represents the derivative of a function. The second derivative of that function is not defined at 1 and 2. Which of these values is an inflection point? Justify your answer.

9.) The top of a 25 foot ladder is sliding down a vertical wall at a constant rate of 4 feet per minute. When the top of the ladder is 15 feet off the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) -3 feet per minute (B) 3 feet per minute (C) 4 feet per minute

(D) -4 feet per minute (E) 5 feet per minute

10.) Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be the function whose derivative is given by $g'(x) = e^{-2x}\big(3f(x) + 2f'(x)\big)$ for all x.

a.) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

b.) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

c.) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

d.) Show that $g''(x) = e^{-2x}\big(-6f(x) - f'(x) + 2f''(x)\big)$.

11.) Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$
(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6
(C) $-1 \le f(x) \le 3$ for all x between -3 and 6
(D) $f(c) = 1$ for at least one c between -3 and 6
(E) $f(c) = 0$ for at least one c between -1 and 3

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

12.) A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

13.) What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

14.) Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x) * g(x)$ and $h'(x) = f(x) * g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

Need a hint?

1.) Is acceleration the first or the second derivative of position? Knowing this will narrow down your answers a lot.

2.) You should rewrite $\frac{1}{x^2}$ as x^{-2} before taking the derivative.

3.) Doesn't that look like a big product rule? $(x-1)^2 * \sin(x)$

4.) a.) You need to know the equation of a tangent line is $y - y_1 = m(x - x_1)$. They give you two of what you need, and the third looks pretty easy to find.

b.) $f(1.2)$ … doesn't that mean that $x = 1.2$?

5.) Deriving an equation with x and y at the same time? That must be implicit! Don't forget that the middle term is $3x * y$.

6.) Extrema are critical points. What's the derivative of a critical point?

7.) Do the two sides of this graph connect at $x = 0$? What do you know about the derivative?

8.) a.) Notice that the integral (area) starts at 1 both times. You need to find area from there to 4 and from there to -1.

b.) Isn't this the same as #6?

c.) What's the derivative of an inflection point? Better go back to that table.

9.) What shape are we talking about here? Remember that one of the three lengths is not changing.

10.) a.) Tangent lines! Same as #4a, except that they gave you all the values.

b.) Is there such a thing as a saddle inflection point?

c.) Looks like you just need to plug in 0 because the rest of the information is given. This is less complicated than it looks.

d.) Doesn't that look like another big product rule? $e^{-2x} * \left(3f(x) + 2f'(x)\right)$

11.) Look up the IVT if you need to. Keep track of what value is an x and what value is a y.

12.) We just learned this! No hints.

13.) A function is increasing in between its critical points.

14.) Even though they gave you $h'(x)$ you should find it yourself. You will notice a difference that will point you towards what $f(x)$ must equal.

Still need more help? Email me or go to this website:

http://www.khanacademy.org/math/calculus/differential-calculus

Answers:

1.) C
2.) C
3.) D
4.) a.) $y - 4 = \frac{1}{2}(x - 1)$
b.) 4.1
5.) A
6.) A
7.) C
8.) a.) $2.5, -4$
b.) $x = -1$
c.) $x = 1$
9.) B
10.) a.) $y = -3x + 2$
b.) No
c.) $y = 4$
11.) D
12.) B
13.) C
14.) E