Practice Time!!!!! ☺

Translate the following to calculus notation.

1. The volume of a cylinder is increasing at a rate of 6 cubic feet per second.

2. The depth of water in a rectangular pool is decreasing at a rate of 2 inches per hour.

3. At rush hour, the number of vehicles going through the SunPass toll booth is increasing a rate of 85 vehicles per hour.

4. Air is escaping from a spherical balloon at the rate of 15 cubic centimeters per second.

Finding a Derivative **In Exercises 27–40, find the derivative of the algebraic function.**

27. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

28. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

29. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right)$

30. $f(x) = x^4\left(1 - \dfrac{2}{x + 1}\right)$

31. $f(x) = \dfrac{3x - 1}{\sqrt{x}}$

32. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

33. $h(s) = (s^3 - 2)^2$

34. $h(x) = (x^2 + 3)^3$

35. $f(x) = \dfrac{2 - \frac{1}{x}}{x - 3}$

36. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right)$

37. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

38. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

39. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, c is a constant.

40. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$, c is a constant.

When a small change in x produces a large change in the value of the function f(x), we say that the function is relative sensitive to change in x and we measure this through f'(x). The larger the value of f'(x) the more sensitive is f(x) to the change of x.

Examples

Non-calculator

It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of the fluid in the tank t hours after the valve is opened is given by the formula

$y = 6(1 - \frac{t}{12})^2$ m $\quad y = 6\left(1 - \frac{t}{6} + \frac{t^2}{144}\right) = 6 - t + \frac{t^2}{24}$

Answer the following questions:

1. What is the domain that makes sense for this situation? Explain.

2. Calculate $\frac{dy}{dt}$ $\quad \frac{1}{12}t - 1$

3. Determine the following values

$\frac{d}{dt}y(0) = -1$

$\frac{d}{dt}y(6) = -\frac{1}{2}$

$\frac{d}{dt}y(12) = 0$

What do these values represent for this situation?

the rate of change of the depth of fluid in tank at different times after the valve is opened.

4. When is y more sensitive to the change in time? Explain.

Right after the valve is opened.

5. Describe the behavior of y in relation to signs and values of $\frac{dy}{dt}$. Check with your calculator.

$$C_{avg(x)} = \frac{C(\cancel{x}+h) - C(x)}{h}$$

$$\text{marginal cost} = \frac{d\,C(x)}{dx}$$

$C(x) = \text{Cost} = 2000 + 100x - 0.1x^2$

If c(x) is the cost production, then the **average cost** for producing h items is given by $\frac{c(x+h)-c(x)}{h}$ and the **marginal cost** of production is the rate of change of cost with respect to the level of production.

If c(x) represent the dollar cost for producing x washing machines and is given by $c(x) = 2000+100x-0.1x^2$, answer the following questions

1. What is the average cost for producing 100 h washing machines?

$$C_{avg} = \frac{C(x+100) - C(x)}{100}$$

2. What is the function that represents the marginal cost for producing x washing machines?

3. What is the marginal cost for producing 2o washing machines, and what is the marginal cost for producing 100 washing machines?

4. When is the cost more sensitive to the change in number of washing machines produced?

1. $\frac{dV}{dt} = 6\ ft^3/sec.$

3. $\frac{dN}{dt} = 85$ vehicles/h

2. $\frac{dh}{dt} = -2\ in/h.$

4. $\frac{dV}{dt} = -15\ cm^3/sec.$

27. $\frac{3}{(x+1)^2}$

28. $f(x) = \frac{x^2+5x+6}{x^2-4} = \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$

$f'(x) = \frac{(x-2)\cdot 1 - (x+3)\cdot 1}{(x-2)^2} = \frac{\cancel{x}-2-\cancel{x}-3}{(x-2)^2} = \frac{-5}{(x-2)^2}$

29. $f(x) = x\left(1 - \frac{4}{x+3}\right) = x - \frac{4x}{x+3}$

$f'(x) = 1 - \frac{(x+3)\cdot 4 - 4x}{(x+3)^2} = 1 - \frac{\cancel{4x}+12-\cancel{4x}}{(x+3)^2} = 1 - \frac{12}{(x+3)^2}$

$= \frac{x^2+6x-3}{(x+3)^2}$

30. $f(x) = x^4\left(1 - \frac{2}{x+1}\right) = x^4 - \frac{2x^4}{x+1}$

$f'(x) = 4x^3 - \frac{8x^3(x+1) - 2x^4}{(x+1)^2} = 4x^3 - \frac{8x^4+8x^3-2x^4}{(x+1)^2}$

$f'(x) = 4x^3 - \frac{6x^4+8x^3}{(x+1)^2} = \frac{-2x^4+4x^3}{(x+1)^2}$

31. $f(x) = \frac{3x-1}{\sqrt{x}}$ $\quad f'(x) = \frac{x^{\frac{1}{2}}(3) - (3x-1)\cdot\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$$f'(x) = \frac{x^{\frac{1}{2}}3 - \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{\frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{x^{-\frac{1}{2}}(3x+1)}{2x}$$

or

$$= \frac{3x+1}{2x^{3/2}}$$

$f(x) = 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} = \frac{3}{2x^{1/2}} + \frac{1}{2x^{3/2}} = \frac{3x+1}{2x^{3/2}}$

32. $f(x) = \sqrt[3]{x}\left(\sqrt{x}+3\right) = x^{\frac{1}{3}}\left(x^{\frac{1}{2}}+3\right) = x^{\frac{7}{6}} + 3x^{2/3}$

$f'(x) = \frac{7}{6}x^{\frac{1}{6}} + 3\cdot\frac{2}{3}x^{-\frac{1}{3}} = \frac{7}{6}x^{1/6} + \frac{2}{x^{1/3}}$

33. $h(s) = (s^3-2)^2 = s^6 - 4s^3 + 4$

$h'(s) = 6s^5 - 12s^2 = 6s^2(s^3-2)$

34 $h(x) = (x^2+3)^3 = x^6 + 9x^4 + 27x^2 + 27$

$h'(x) = 6x^5 + 36x^3 + 54x = 6x(x^4 + 6x^2 + 9) = 6x(x^2+3)^2$

35. $f(x) = \frac{2-\frac{1}{x}}{x-3} = \frac{2x-1}{x^2-3x}$

$$f'(x) = \frac{2(x^2-3x) - (2x-1)\cdot(2x-3)}{(x^2-3x)^2} = \frac{2x^2 - \cancel{6x} - 4x^2 \cancel{+6x} + 2x - 3}{(x^2-3x)^2}$$

$$= \frac{-2x^2+2x-3}{x^2(x-3)^2}$$

36. $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right) = 2x - \frac{x^2}{x+1}$

$$g'(x) = 2 - \frac{2x(x+1) - x^2}{(x+1)^2} = 2 - \frac{2x^2 + 2x - x^2}{(x+1)^2} = 2 - \frac{x(x+2)}{(x+1)^2}$$

37. $f(x) = (2x^3+5x)(x-3)(x+2)$

$$f'(x) = (6x^2+5)(x-3)(x+2) + (2x^3+5x)(x+2 + x-3)$$

$$= (6x^2+5)(x-3)(x+2) + (2x^3+5x)(2x-1)$$

$$= (6x^2+5)(x^2-x-6) + (2x^3+5x)(2x-1)$$

$$= \cancel{6x^4} - \cancel{6x^3} - \cancel{36x^2} + \cancel{5x^2} - 5x - 30 + \cancel{4x^4} - \cancel{2x^3} + \cancel{10x^2} - 5x$$

$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

38. $f(x = (x^3-x)(x^2+2)(x^2+x-1)$

$$f'(x) = (2x+1)(x^3-x)(x^2+2) + (x^2+x-1)\left((3x^2-1)(x^2+1) + (x^3-x)\,2x\right)$$

39. $$\frac{2x(x^2-c^2)-(x^2+c^2)(2x)}{(x^2-c^2)^2} = \frac{2x(\cancel{x^2}-c^2-\cancel{x^2}-c^2)}{(x^2-c^2)^2} = \frac{-4cx^2}{(x^2-c^2)^2}$$

40 $$f(x) = \frac{c^2-x^2}{c^2+x^2}$$

$$f'(x) = \frac{-2x(c^2+x^2) - (c^2-x^2)\cdot 2x}{(c^2+x^2)^2}$$

$$= \frac{-2x(c^2+\cancel{x^2}+c^2-\cancel{x^2})}{(c^2+x^2)^2} = -\frac{2cx}{(c^2+x^2)^2}$$